THE STUDY OF NEW TYPE OF RF ACCELERATION IN SCALING FFAG ACCELERATOR

Emi Yamakawa*[A)], Thomas Planche [A)], Jean-Baptiste Lagrange[A)],
Tomonori Uesugi[B)], Yasutoshi Kuriyama[B)], Yoshihiro Ishi[B)], Yoshiharu Mori[B)],
Izumi Sakai[C)], Kota Okabe[C)], Makoto Inoue[C)],
[A)]Graduate School of Engineering, Kyoto University, Kyoto, Japan
Katsura Nishikyo-ku Kyoto, 615-8530, Japan
[B)]Kyoto University Research Institute, Kumatori, Japan
Asashiro-Nishi, Kumatori-cho, Sennan-gun, Osaka, 590-0458, Japan
[C)]Fukui University, Fukui, Japan
Bunkyo, Fukui-shi, Fukui, Japan

Abstract

High power proton driver to produce intense secondary particle beams is required for various fields. Fixed-field alternating gradient (FFAG) accelerator is one of the possible candidates for such proton drivers. In order to produce much higher intensity proton beams in scaling type of FFAG, a new type of acceleration scheme, called serpentine acceleration, is considered in this paper. The longitudinal hamiltonian for serpentine acceleration is derived analytically. The application for proton driver, based on serpentine acceleration, is also presented.

INTRODUCTION

Particle accelerators have been widely used not only for particle physics[1] but also for many applications such as cancer therapy[2] [3] and nuclear power engineering[4] etc. For these purposes, requests for high power proton drivers to produce intense secondary particle beams, in particular, are increasing. Fixed-field alternating gradient (FFAG) accelerators[5] are one of the possible candidates for such proton drivers. In FFAG, the guiding magnetic field is static. For this advantage, the repetition rate of acceleration only depends on the capability of rf system.

Various methods of beam acceleration using rf cavities have been used and proposed for FFAG. One of them is an ordinary method where the beam acceleration is realized with frequency modulation of the rf system. With this acceleration scheme, the repetition rate of acceleration is limited by the capabilities of the rf system such as its changing speed of rf frequency and voltage. Another scheme is the stationary bucket acceleration[6] where rf frequency is fixed. Since cw operations are possible in stationary bucket acceleration, higher current beams can be obtained. However, the total energy gain is limited within a stationary bucket. In order to increase it, a new type of acceleration scheme, called serpentine acceleration[7], has been proposed. If serpentine acceleration is used in the scaling type of FFAG, the limitation of total energy gain within a stationary bucket can be dissapeared even if constant rf frequency is used.

* yamakawa@post3.rri.kyoto-u.ac.jp

In this paper, the longitudinal hamiltonian for the stationary bucket acceleration in the scaling FFAG is derived analytically, and application for proton driver, based on serpentine acceleration in scaling FFAG, are also presented.

FEATURES OF SCALING FFAG

In cylindrical coordinates, the magnetic field in scaling FFAG is given by

$$B_z(r, z = 0) = B_0 \left(\frac{r}{r_0} \right)^k, \qquad (1)$$

where r is the radial coordinate with respect to the center of the ring. B_0 is the magnetic field at the reference radius r_0. k is the geometric field index and z is the vertical coordinate. Magnetic fields of the scaling FFAG magnets are kept constant during particle acceleration. For this reason, rapid acceleration with high repetition rate is possible. Also zero chromaticity is achieved by using non-linear magnetic field which is expressed by Eq.(1). It makes a betatron tune constant even if particle momentum changes. Therefore stable beam acceleration can be realized.

SERPENTINE ACCELERATION IN SCALING FFAG

The principle of serpentine acceleration is to accelerate a beam between stationary buckets and a beam passes through the transition energy during acceleration. The constant rf frequency is adopted in the serpentine acceleration. Therefore cw operation can be achieved.

Longitudinal Hamiltonian in scaling FFAG

In order to examine features of serpentine acceleration, longitudinal hamiltonian for the stationary bucket acceleration in scaling FFAG is derived analytically.

The closed orbits for different momentum (P) is given by

$$r = r_0 \left(\frac{P}{P_0}\right)^{\frac{1}{k+1}}, \tag{2}$$

where P_0 is the reference momentum at r_0.

With constant rf frequency in the scaling FFAG, the longitudinal phase discrepancy per revolution $\Delta\phi$ is written by

$$\Delta\phi = 2\pi(f_{rf} \cdot T - h), \tag{3}$$

where h is the harmonic number, f_{rf} is the rf frequency and T is the revolution period of a non-synchronous particle. Equation(3) becomes

$$\frac{\Delta\phi}{2\pi} = \frac{hT}{T_s} - h, \tag{4}$$

where T_s is the revolution period of a synchronous particle. Equation(4) is also expressed with another description based on Eq.(2) as follows;

$$\begin{aligned} \frac{T}{T_s} &= \left(\frac{r}{r_s}\right) \Big/ \frac{P/E}{P_s/E_s} \\ &= P_s^{1-\alpha} \frac{E}{E_s} P^{\alpha-1}, \end{aligned} \tag{5}$$

where r_s is the reference radius, α is the momentum compaction factor and E_s is the reference energy at the reference radius. As shown in Fig.1, the two stationary energies, E_{s1} and E_{s2}, close to each other when rf frequency is near the revolution time of transition energy. Combining Eq.(4) and Eq.(5), the phase difference becomes

$$\Delta\phi = 2\pi h\Big(\frac{P_s^{1-\alpha}}{E_s} E P^{\alpha-1} - 1\Big). \tag{6}$$

Now we exchange $\Delta\phi/2\pi$ and $d\phi/d\theta$ to derive the phase and energy equation of longitudinal motion,

$$\frac{d\phi}{d\theta} = h\Big(\frac{P_s^{1-\alpha}}{E_s} E P^{\alpha-1} - 1\Big) \tag{7}$$

$$\frac{dE}{d\theta} = \frac{eV_{rf}}{2\pi} \sin\phi, \tag{8}$$

where V_{rf} is the rf voltage per turn and θ is an azimuthal angle in the machine. We introduce the energy variable E canonically conjugate to the coordinate variable ϕ. Equation(7) and (8) derive the longitudinal hamiltonian:

$$H(E, \phi; \theta) = h\Big(\frac{1}{\alpha+1} \frac{P^{\alpha+1}}{E_s P_s^{\alpha-1}} - E\Big) + \frac{eV_{rf}}{2\pi} \cos\phi. \tag{9}$$

Longitudinal Phase Space

The features of serpentine acceleration are examined by hamiltonian (Eq.(9)). As shown in Fig.2, when the two stationary energies are far from each other, the two stationary buckets are also separated. When the two stationary energies close to each other, however, a channel between the two stationary buckets appears as shown in Fig.3. If particles can be accelerated using this channel, total energy gain through the acceleration becomes larger than the total energy gain within a stationary bucket.

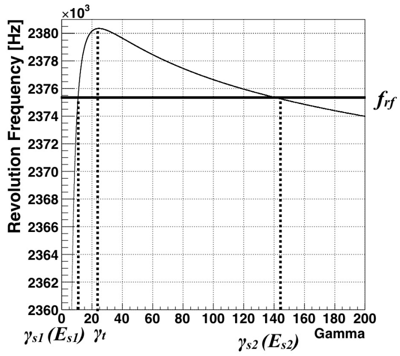

Figure 1: Relation between revolution frequency and energy of particles, where γ_t is the transition γ. E_{s1} is the stationary energy which is lower than the transition energy. E_{s2} is the other stationary energy which is higher.

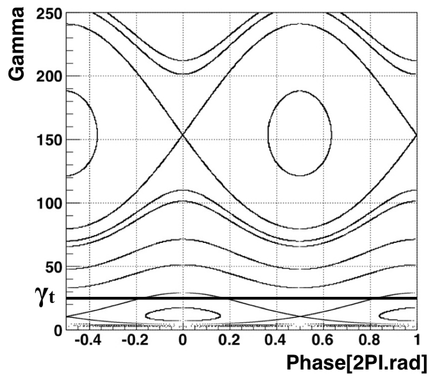

Figure 2: Longitudinal phase space when synchronous energies are far from the transition energy.

Minimum rf Voltage

The minimum rf voltage to make serpentine acceleration scheme is derived from Eq.(9). Since the separatrix goes through two unstable fixed points as shown in Fig.4, the relation between $H(E_{s1}, \pi)$ and $H(E_{s2}, 0)$ is

$$H(E_{s1}, \pi) = H(E_{s2}, 0). \tag{10}$$

The relation between E_{s1} and E_{s2}, which are shown in Fig.1, is

$$E_{s1} P_{s1}^{\alpha-1} = E_{s2} P_{s2}^{\alpha-1}. \tag{11}$$

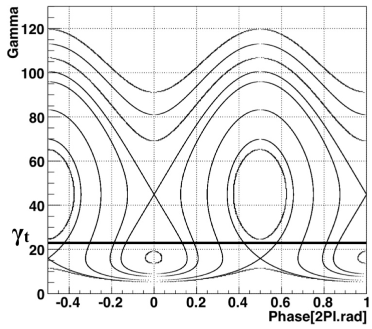

Figure 3: Longitudinal phase space near the transition energy. The two stationary buckets are close each other.

From Eq.(10) and Eq.(11), the minimum rf voltage to make serpentine acceleration is derived;

$$V_{rf} = \pi h \left[\frac{1}{\alpha+1} \left(\frac{P_{s1}^2}{E_{s1}} - \frac{P_{s2}^2}{E_{s2}} \right) + (E_{s2} - E_{s1}) \right]. \quad (12)$$

Equation(12) shows that once the k value and the stationary energy are given, the minimum rf voltage to achieve serpentine acceleration can be calculated.

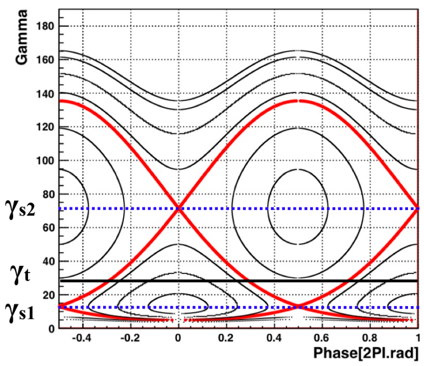

Figure 4: Longitudinal phase space when the separatrix goes through the two unstable fixed points. γ_{s1} corresponds to E_{s1} and γ_{s2} corresponds to E_{s2}.

APPLICATION

Proton driver is considered with serpentine acceleration. Parameters of proton driver are summarized in Table.1. The beam trajectories in the longitudinal phase space are shown in Fig.5. As shown in Fig.5, injection kinetic energy is 500 MeV$\pm$1%. Injection phase range is from 0.25 to 0.3 2πrad. Final kinetic energy is 2018 MeV$\pm$1.4%. The number of turn during acceleration is 35 turns. The ratio of injection to final momentum is 2.6. Furthermore, the injected beam shape is rotated in phase space.

Table 1: The parameters of proton accelerator

Stationary kinetic energy	650 MeV
Mean radius (γ_s)	10 m
k value	3
Harmonic number h	1
rf voltage / turn	50 MV
rf frequency	3.7 MHz

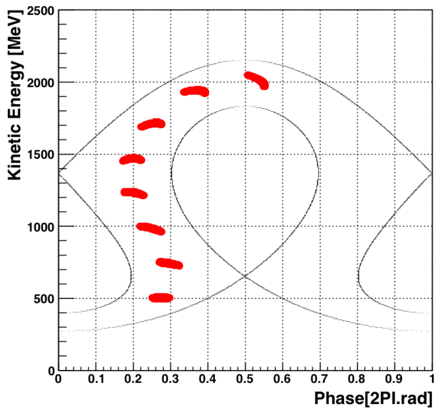

Figure 5: Proton beam is plotted by every 5 turns in longitudinal phase space. When 500 MeV proton beam is injected, it can be finally accelerated to 2 GeV in this scheme.

CONCLUSION

In order to obtain a high power proton beam with high repetition rate, serpentine acceleration has been proposed for the scaling FFAG. The longitudinal hamiltonian in the scaling FFAG has been obtained analytically. By using longitudinal hamiltonian, the features of serpentine acceleration have been examined . For further study of serpentine acceleration in scaling FFAG, some experiments with real machine should be done.

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