DEVELOPMENT OF A TRACKING CODE FOR A SMALL STORAGE RING

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Abstract

A tracking code based on C++ language has been developed for HiSOR-II which is under designing for the future plan at Hiroshima Research Institute for Synchrotron Radiation Science. If the radiation effect is taken into consideration, the electron motion in a ring is non-symplectic. Therefore, the explicit Runge-Kutta method can be applied in numerical tracking in electron storage ring. In addition to general magnet models calculated from Hamiltonian, an element model based on Runge-Kutta method is developed, which can take the 3D magnetic field into calculation. Because magnet components of HiSOR-II are combined-function magnets, the adjustability of sextupole components is limited. We want to use the magnetic field in longitudinal direction produced by the edge effect to estimate chromaticity, which will be compared with the Maxwellian fringe field effect. This program can perform a 6D tracking updated from a previous Runge-Kutta tracking code. The ring optics parameters such as closed orbit, Twiss parameter and tune can also be obtained.

INTRODUCTION

A new low-energy storage ring HiSOR-II is under designing for the increasing demand of high-brilliance radiation at Hiroshima Research Institute for Synchrotron Radiation Science. One proposed ring lattice layout is shown in Fig. 1, which only consists of two kinds of magnet. The bending magnet and quadrupole magnet are both combined-function magnets, which can reduce the operation cost, the accelerator size and the amount of materials for the lattice components.

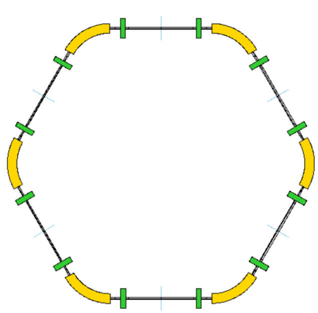

Figure 1: Layout of HiSOR-II ring.

In the ring design, it is found that the energy-dependent tune shift will change due to the existence of dipole fringe field effect, which is explained in the next section. The sextupole components to correct the chromaticity are different in each case. Because magnet components of HiSOR-II are combined-function magnets, the adjustability of sextupole components is limited. To estimate chromaticity correctly and determine the magnet parameters, a tracking code based on C++ language has been developed, which can take 3D magnetic field into calculation and calculate optics parameters.

FRINGE FIELD EFFECT OF DIPOLE MAGNET

Magnetic Field at the Dipole Edge

It is assumed the dipole magnet has an infinitely parallel face. The magnetic flux distribution is shown in Fig. 2. Inside the magnet, the magnetic field is a constant value $B_0\vec{\mathbf{y}}$. Therefore, the magnetic vector potential (A_x,A_y,A_z) is $(0,0,-xB_0)$. Around the edge of the magnet where $z = 0$, the magnetic field is not constant any more.

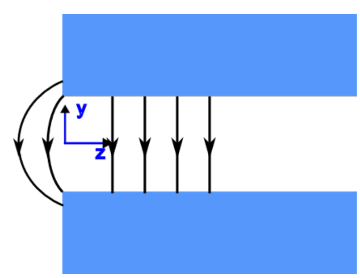

Figure 2: Magnetic flux at the edge of a dipole magnet.

To approximately describe the fringe field effect, one method is to use a step function for B_y [1], and the magnetic fields are given by

$$\begin{aligned}
B_x &= 0 \\
B_y &= B_0 u(z) - \frac{y^2}{2}\delta'(z) + B_0\frac{y^4}{4!}\delta'''(z) - \ldots \\
B_z &= yB_0\delta(z) - B_0\frac{y^3}{3!}\delta''(z) + B_0\frac{y^5}{5!}\delta'''''(z) - \ldots
\end{aligned} \tag{1}$$

where z=0 is at the entrance of dipole magnet. It can be examined that the magnetic fields satisfy Maxwell's equations.

Contribution to the Chromaticity

If an electron comes to the entrance of the magnet, the magnetic field at $z = 0$ can be used to calculate the change of momentum, which is the Maxwellian fringe field effect [2]. The momentum P_x in x direction stays the same, and the

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momentum P_y in y direction changes due to the fringe field. The particle motions after the fringe field at the entrance are calculated as

$$\begin{aligned} x^f &= x + \frac{y^2}{2\rho P_z} \\ y^f &= y \\ P_x^f &= P_x \\ P_y^f &= P_y - \frac{y}{\rho}\frac{P_y}{P_z} \end{aligned} \tag{2}$$

where P_z is the momentum in z direction, ρ is radius of curvature, the superscript f means the motion after the fringe field effect. It indicates that the dipole fringe field produces a vertical focusing effect. Therefore, the tune in vertical direction will be affected if there is a closed orbit distortion due to an energy deviation.

To examine the fringe field effect, the tune of HiSOR-II lattice is calculated using the C++ program. Particle motions in drift, quadrupole and dipole are derived from a general Hamiltonian, which is given by

$$\begin{aligned} &H(x, p_x, y, p_y, \delta, \tau; s) = \\ &-(1+\frac{x}{\rho})\sqrt{(1+\delta)^2-(p_x-a_x)^2-(p_y-a_y)^2} \\ &-(1+\frac{x}{\rho})a_z \end{aligned} \tag{3}$$

where $(x, p_x, y, p_y, \delta, \tau; s)$ are canonical variables, p_x and p_y are normalized momentum, δ is energy deviation, and τ is pathlength. $a_{x,y,z}$ are normalized vector potentials.

The energy dependent tune shift is calculated with and without the dipole fringe field effect, which is shown in Fig. 3.

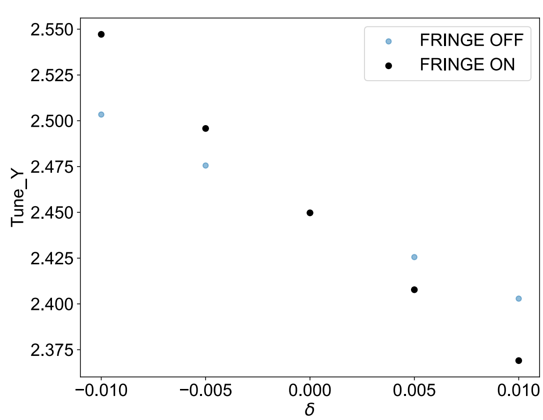

Figure 3: Different Energy-dependent tune shift due to the fringe field effect.

In electron storage ring, sextupole magnet is used to compensate the tune shift caused by energy deviation. It represents that the designed sextupole components strength is related to the model that we use. The adjustability of sextupole components is limited due to the lattice design. In order to estimate the chromaticity correctly to decide the strength of sextupole components, we want to use a real magnetic field to replace the fringe field model to calculate optics parameters.

PARTICLE TRACKING BY 3D MAGNETIC FIELD

As for the tracking method of the initial C++ program, the symplectic integration by Hamiltonian is used to conserve phase space exactly. However, this method is hard to use when B_x, B_y, and B_z are are present.

The equation of motion from Lagrangian is able to treat 3D magnetic field easily [3]. The equations of motion for an ultra-relativistic particle are transformed to a simple form as follows.

$$\begin{aligned} x'' &= \frac{D}{a(1+\delta)}\left[ay'B_z + x'y'B_x - (x'^2+a^2)B_y\right] \\ &\quad + \frac{1}{\rho a}(2x'^2+a^2) \\ y'' &= \frac{D}{a(1+\delta)}\left[-ax'B_z - x'y'B_y + (y'^2+a^2)B_x\right] \\ &\quad + \frac{1}{\rho a}(2x'y') \\ s' &= \sqrt{x'^2+y'^2+a^2} \\ a &= 1+\frac{x}{\rho} \\ D &= \frac{e}{p}\sqrt{x'^2+y'^2+(1+\frac{x}{\rho})^2} \end{aligned} \tag{4}$$

where ρ is the radius of curvature, p is the momentum, and $'$ donates the differentiation with s.

If the radiation effect is taken into consideration, the electron motion in a ring is non-symplectic. Therefore, the explicit Runge-Kutta method can be applied to track electron by using adjusted step sizes. A dipole magnet model is added to perform a 6D tracking updated from a previous Runge-Kutta tracking code [4].

CODE IMPLEMENT

A Magnet Model Based on 3D Magnetic Field

As for a dipole magnet that has a normal quadrupole component, the magnetic fields satisfy the equations $B_y(y) = B_y(-y)$, $B_x(y) = -B_x(-y)$, and $B_z(y) = -B_z(-y)$. Therefore, the magnetic field expansions can be given by

$$\begin{aligned} B_y(s) &= \sum_{m,n=0}^{\infty} y^{2m}x^n a(s)_{m,n} \\ B_x(s) &= y\sum_{m,n=0}^{\infty} y^{2m}x^n b(s)_{m,n} \\ B_z(s) &= \sum_{m,n=0}^{\infty} y^{2m}x^n d(s)_{m,n} \end{aligned} \tag{5}$$

where $a(s)_{m,n}$, $b(s)_{m,n}$, and $d(s)_{m,n}$ are strength coefficients.

To obey Maxwell equation $\nabla \times \mathbf{B} = 0$, the coefficients are satisfied by the following equations.

$$\begin{aligned} a'_{m,n} &= (2m+1)(d_{m,n} + d_{m,n-1}/\rho) \\ (n+1)a_{m,n+1} &= (2m+1)b_{m,n} \\ b'_{m,n} &= (n+1)(d_{m,n+1} + d_{m,n}/\rho) \end{aligned} \tag{6}$$

Therefore, the coefficients of the polynomial of $B_z(s)$ can be obtained from the expression of transverse magnetic field. In the code, the coefficient of B_z is up to sextupole component.

Symplectify Transfer Matrix

To calculate the optics parameters, the transfer matrix of the magnet that has 3D magnetic field are calculated. Application of a differential algebra package is a direct method to get transfer matrix from tracking. However, such a package is not developed for this program now. To get the matrix of magnet, the components of transfer matrix is derived by tracking particle around closed orbit. The deviation is very small, and the value is 1×10^{-7} for each variable.

Because the tracking method is non-symplectic, the transfer matrix is nearly symplectic. For the convenience of calculation, the matrix is symplectified by a simple iteration as follows [5].

$$M = \frac{1}{2}(3I - MSM^TS^T)M \tag{7}$$

where the matrix S is given by

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \tag{8}$$

Optics Parameters Calculation

To examine the calculation by the code, the Twiss parameters of one cell in HiSOR-II lattice are calculated by different models, which is shown in Fig. 4. The solid lines are beta and dispersion functions calculated by equation of motion from Hamiltonian. Because we do not have magnetic field data of a sector magnet, a hard-edge magnet model is used for the calculation. That is to say, the magnet only has transverse field components, and the field strength is equivalent to that of the previous model using Hamiltonian. For the tracking by Runge Kutta method, the matrix is not calculated in each slice of magnet. Therefore, the Twiss parameter is only calculated at the entrance of each element.

The Twiss parameter calculated by Eq. (4) is consistent with the result using equation of motion from Hamiltonian. As for the tune calculated by Hamiltonian, the tune in horizontal and vertical are 2.74998 and 2.44971, respectively. The tune calculated by Eq. (4) are 2.75006 and 2.44919, respectively.

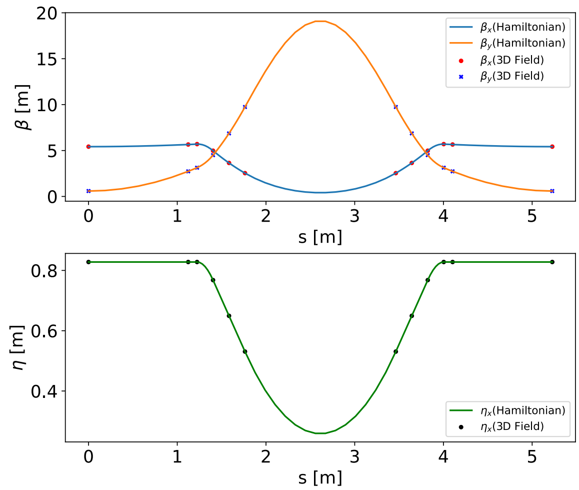

Figure 4: Twiss parameters calculated by different models.

CONCLUSION

To evaluate chromaticity in a small ring, a code has been developed based on C++ and can calculate particle motion by equation of motions from Hamiltonian and 3D magnetic field.

In the future, a magnetic field data from a sector magnet will be loaded to compute optics parameters. The result will be compared with the simulation result in Fig. 3.

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