PIC Simulations of SASE FELs

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[Abstract]

A one-dimensional PIC simulation of SASE FELs is developed. In the code, the electron bunch is divided into many cells with the ponderomotive wavelength. The evolution of the radiation pulse is studied. The simulation results are compared with steady-state FELs.

(1) Introduction

The free electron laser (FEL) based on self-amplified spontaneous emission (SASE) is achieved in a single pass of an electron bunch through a very long undulator. The electron beam amplifies exponentially an initially existing radiation field under the FEL resonance condition that the radiation wave must travel one radiation wavelength λ further than the electron during the time that the electron travels one wiggler period λw. Therefore the electron slips a distance $Nw\lambda$ relative to the leading edge of the radiation pulse, where Nw is the number of periods in the undulator. In SASE FELs, the slippage distance $Nw\lambda$ is not negligibly small relative to the beam length. So the slippage between the radiation pulse and the electron beam is important in a simulation of the SASE FEL.

(2) Basic equations

The motion of the simulation electron (i=1,2,3.....n) is expressed by the following first-order equations:

$$\frac{\partial \phi_i(\eta,\tau)}{\partial \tau} = p_i(\eta,\tau), \tag{1}$$

$$\frac{\partial p_i(\eta,\tau)}{\partial \tau} = -\left[A(\eta,\tau)\exp\left(i\phi(\eta,t)\right) + c.c.\right], \tag{2}$$

where the following dimensionless variables are introduced:

$$p_i = \frac{\gamma_i - \gamma_r}{\rho\gamma_r}; \tau = 2\rho k_W ct; \eta = 2\rho k_W\left(z - \bar{\beta}_Z ct\right);$$

$$|A|^2 = \frac{1}{\rho}\frac{\frac{\varepsilon_0}{4}|E|^2 \cdot \lambda\Sigma}{N\gamma_r mc^2}.$$

The dynamical variables are the phase ϕ and the relative energy deviation p. ϕ is the conjugate variable to p. τ and η are scaled coordinates. τ is the normalized distance from the undulator entrance. η is the electron coordinate with respect to the beam center. The radiation field is represented by a complex amplitude A. The quantity ρ is known as the FEL parameter.

The field equation under the slowly varying envelope approximation can be written in complex form as follows:

$$\left[\frac{\partial}{\partial \tau} + v\frac{\partial}{\partial \eta}\right]A(\eta,\tau) = \frac{1}{n}\sum_{l=1}^{n}\exp\left(-i\phi_l(\eta,\tau)\right), \tag{3}$$

where the sum represent the contribution from all simulation particles within a ponderomotive wavelength.

(3) PIC code

In order to study the radiation profile, we divide the electron bunch into many cells with dimensions given by the ponderomotive wavelength. We use the PIC (particle-in-cell) simulation method.

We numerically solve the FEL equations (1) and (2) in each cell and obtain the generated field in the cell by using the following differential form:

$$dA[k] = \left(\frac{1}{n}\sum_{l=1}^{n}\exp\left(-i\phi_l(\eta,\tau)\right)\right)d\tau. \tag{3a}$$

The velocity slip between the radiation field and the electron can be realized by using the following first-order wave equation to represent the field transfer into the next cell:

$$A[k]_{New} = A[k] + \frac{1}{4\pi\rho}(A[k+1] - A[k]). \quad (3b)$$

Under the new field, the particles are accelerated or decelerated. The field acting on the simulation particles can be obtained to interpolate the field on the neighboring grid:

$$A = A[k] \pm (A[k \mp 1] - A[k]) \frac{\Delta\varphi_i}{\pi}.$$

The movement of the particle generates a new field. Such iterations are continued to the end of the undulator.

(4) Initial condition

In SASE FELs, the initial power source is the shot noise in the beam. The startup from shot noise is modeled by adding random fluctuations to the longitudinal coordinates of the simulated particles. We place the simulated particles near regularly-spaced positon:

$$\phi_i(\eta) = \frac{2\pi i}{n} + x_i.$$

The fluctuation xi has a normal distribution with width Δ:

$$P(x_i) = \frac{1}{\sqrt{2\pi}\Delta} \exp\left(-\frac{x_i^2}{2\Delta^2}\right).$$

We choose n=50 and Δ=0.022 according to ref. (1).

(5) Simulations

The FEL equations (1)~(3) are solved numerically under the assumption that the initial beam has no energy spread. The shape of the initial beam bunch is taken to be parabolic. The charge of the simulated particle has an assigned value which reflects the parabolic profile of the beam. The number of particles in each cell is fixed for numerical stability.

In our simulations, the following values are assumed: the total number of cells is 300, and the particles are loaded in 200 successive cells. In order to save computation time, the number of the particles in each cell is 50. The FEL fundamental parameter ρ equals 0.0239.

As described above, every dτ step, the field equations are solved without the slippage term in each bucket, and then the field is transferred forward.

Figs.1(a)~(d) respectively show the normalized field A at the normalized distance τ=0.1,1,5,12 from the undulator entrance. The spikes seeded by the noise non-uniformity are distributed throughout the radiation pulse around the entrance of the undulator. The envelope of the generated field is approximately parabolic. Gradually, sub-pulse structures appear as the field grow. As can be seen, the generated pulse is composed of a number of spikes which are spaced at a distance related to the slippage length. The field saturates around τ=12.

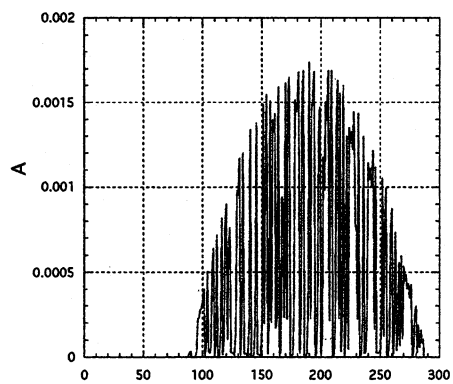

Fig.1(a)

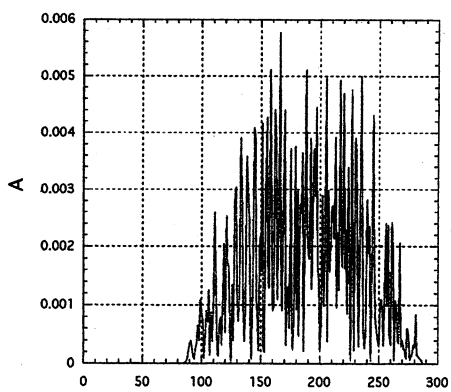

Fig.1(b)

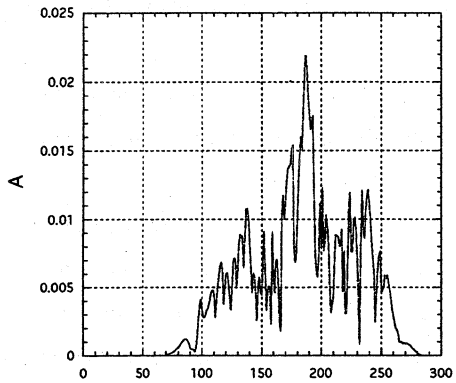

Fig.1(c)

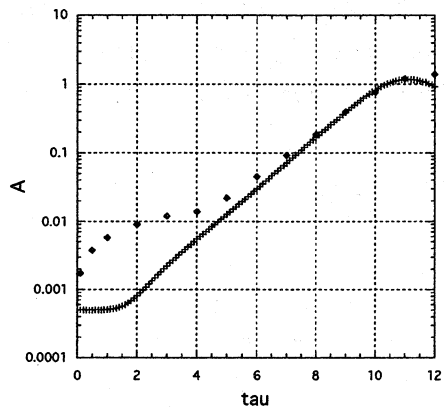

Fig. 2

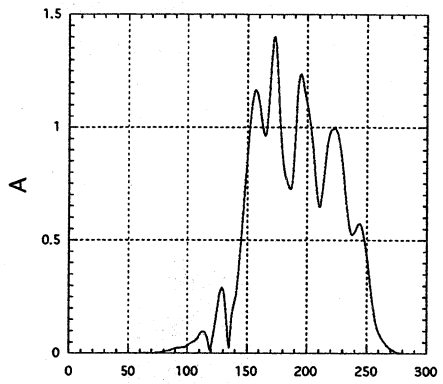

Fig.1(d)

References

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(6) Comparison with steady state FELs

Fig.2 shows the result of the steady state model FEL simulation code with the initial field A=0.0005. The field begins to grow slowly with distance, grows exponentially and saturates.

In the SASE FEL, the development of the peak value of the generated radiation is plotted by the squares on the Fig.2. At the beginning of the undulator, the field grows quickly with distance. In the exponential regime, an excellent agreement is obtained between the steady-state model and the pulsed SASE model.