APPLICATION OF A PAUL TRAP TO THE STUDY OF SPACE-CHARGE-DOMINATED BEAMS I : THEORY

Masao Kanou, Ryota Takai, Yoshio Wada, Kiyokazu Ito, and Hiromi Okamoto
Department of Quantum Matter, Graduate School of Advanced Sciences of Matter, Hiroshima University, 1-3-1 Kagamiyama, Higashi-Hiroshima 739-8530, Japan

Abstract

In this paper, we make discussions on an analogy between linear beam transport channels and a plasma trap system. A radio-frequency quadrupole configuration is considered as the trap. It is demonstrated that the dynamics of a single-species plasma confined in the trap is almost equivalent to that of charged-particle beams propagating through alternating gradient lattices. This implies that one can experimentally investigate various collective behaviors of space-charge-dominated beams by using a compact trap device instead of large accelerators.

1 INTRODUCTION

In recent years, various applications of high-power ion beams, such as the production of tritium, the transmutation of radioactive waste, etc., have been proposed and theoretically examined [1-3]. Since these applications require average currents much higher than those in existing accelerators, extra attention must be paid to the collective instabilities induced by Coulomb self-fields. In fact, even a very small amount of uncontrollable beam loss is enough to activate the whole machine structure. It is thus crucial to have detailed information on the dynamics behavior of intense beam, so that one can design a practically acceptable machine.

Most of previous works on space-charge effects have been either analytical or numerical [4-9]. Analytic studies are always based on some simplifying models since the self-consistent treatment of nonlinear particle interactions is extremely difficult. On the other hand, a number of numerical simulations have been performed with the help of powerful computers to attain better understandings of collective beam motions. However, trustable multi-dimensional simulations are generally quite time-consuming, which eventually forces one to introduce some simplification of reality again. A flexible attitude without adhering to the conventional approaches seems to be desired now for making an essential progress in next-generation beam studies.

Suppose a linear transport channel consisting of a series of quadrupole focusing magnets. The Hamiltonian governing the transverse beam motion is then given by

$$H_{beam} = \frac{P_x^2 + P_y^2}{2} + \frac{1}{2}K_1(s)\left(x^2 - y^2\right) + \frac{e}{p_0\beta_0 c\gamma_0^2}\phi, \qquad (1)$$

where e is the charge state of particles, c is the speed of light, β_0 and γ_0 are the Lorentz factors, and p_0 is kinetic momentum of the beam orbit. Here, the focusing function $K_1(s)$ has been defined by $K_1(s) = -eG/p_0$ with G being the gradient of the quadrupole fields. The scalar potentials ϕ can be derived from the Poisson equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi = -\frac{e}{\varepsilon_0}\iint f dp_x dp_y\,, \qquad (2)$$

where f is the particle distribution function. Provided that the effect of intra-particle collisions is negligible, f satisfies the Vlasov equation

$$\frac{\partial f}{\partial s} + [f, H_{beam}] = 0, \qquad (3)$$

where [,]stands for the Poisson bracket. Equation (1) to (3) clearly form a closed set, thus self-consistently describing the beam motion.

In the following, we compare the dynamics of charged-particle beams with that of one-component plasmas confined in compact trap system. In Section 2, we show that the plasma motion in a radio-frequency quadrupole trap (Paul trap) obeys the equation equivalent to Eq. (1) to (3). This implicates that one can experimentally investigate various features of space-charge-dominated beams by employing the plasma trap. In Section 3, we outline possible beam physics experiments, which can be carried out with Paul trap.

2 BASIC EQUATIONS FOR TRAPPED SINGLE-SPECIES PLASMAS

Assuming the plasma motion to be non-relativistic, the Hamiltonian for the trapped charged particles is given by

$$H_{plasma} = \frac{1}{2m}\left(\vec{p} - e\vec{A}_{ext}\right)^2 + e\left(\phi_{ext} + \phi_{sc}\right), \qquad (4)$$

where m is the rest mass of the particles confined by external potentials $\vec{A} = (A_x^{ext}, A_y^{ext}, A_z^{ext})$ and ϕ_{ext}. The space-charge potential ϕ_{sc} is obtained from the Poisson equation similar to Eq. (2). A schematic view of the Paul trap is illustrated in Fig. 1. The transverse confinement of plasma is accomplished by radio-frequency electric field of quadrupole symmetry while static voltages are applied to two end plates to form a longitudinal potential well. The electrodes facing each other have the same potential as indicated in Fig. 1(b). The signs of the periodic voltage functions $V_x(t)$ and $V_y(t)$ must be opposite, so that a quadrupole focusing field is generated. Assuming simply that $V_x(t) = -V_y(t) = V(t)$ and $U_x = U_y = 0$, we obtain the scalar potential

$$\phi_{ext} \approx \left(\frac{x^2 - y^2}{R^2} \right) V(t), \tag{5}$$

where R is the aperture radius corresponding to the minimum spacing between the longitudinal axis and the electrode poles. Since there is no external magnetic fields, we can put $\boldsymbol{A}$ =0. Then, substitution of Eq. (5) into Eq. (4) yields

$$H_{trap} = \frac{p_x^2 + p_y^2}{2m} + \left(\frac{x^2 - y^2}{R^2} \right) eV(t) + e\phi_{sc}, \tag{6}$$

where we have ignored the longitudinal motion as it can be separated from the transverse dynamics. Performing scale changes according to

$$\frac{H_{trap}}{mc^2} \rightarrow H_{trap}, \frac{p_x}{mc} \rightarrow p_x, \frac{p_y}{mc} \rightarrow p_y, \tag{7}$$

Eq. (6) is reduced to

$$H_{trap} = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} K_2(z)\left(x^2 - y^2\right) + \frac{e}{mc^2 \phi_{sc}}, \tag{8}$$

where $K_2(z) = 2eV(z)/mc^2R^2$, and the scaled independent variable is $z = ct$. In most applications, $V(z)$ is a sinusoidal function ; namely, $V(z) = \cos(2\pi z/\lambda)$ where λ is the wavelength of radio-frequency field.

Analogous to the case of charged-particle beam, the distribution function f_p for the plasma obeys the Vlasov equation

$$\frac{\partial f_p}{\partial s} + \left[f_p, H_{trap} \right] = 0, \tag{9}$$

where ϕ_{sc} is a solution to the Poisson equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \phi_{sc} = -\frac{e}{\varepsilon_0} \iint f_p dp_x dp_y, \tag{10}$$

Since H_{trap} has the form identical to H_{beam} except for the coefficients, the nonlinear dynamics described by Eqs. (9) and (10) is physically equivalent to that by Eqs. (2) and (3) [10].

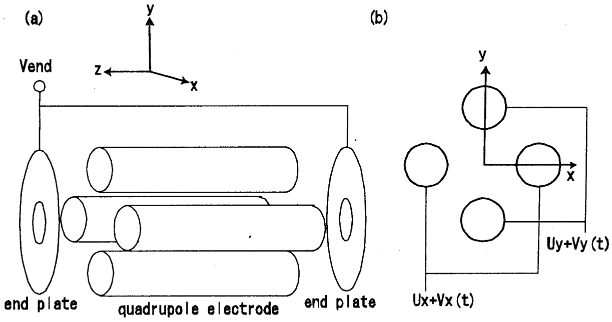

Fig.1 Schematic drawing of a linear ion trap

3 POSSIBLE EXPERIMENTS

As mathematically verified in the last section, the plasma trap can reproduce actual situations in linear beam transport channels. This fact gives us a unique opportunity of making systematic experimental researches in space-charge-dominated beam physics without relying on complex accelerators. Trap-based experiments clearly have many advantages. For instance, the trap is much more compact and cheaper than accelerator-based experimental systems and is free from radioactivation due to particle loss. Furthermore, the detailed information of particle distribution can be obtained very easily as the plasma centroid is at rest in the laboratory frame. Even the time-evolution of the distribution is observable as discussed below.

The space-charge effects particularly important from a practical point of view are "beam halo formation" and "coherent resonances". By adjusting the z-dependence of $V(z)$ to the variation pattern of G along a beam transport line, the dynamical systems governed by the Hamiltonians (1) and (8) become physically identical. Figure 2 displays an example of the voltage function $V(z)$ replicating a simple FODO lattice. The pulse height V_0 is properly chosen such that the phase advance of single-particle oscillation per unit variation becomes the same as that of betatron motion per FODO cell. It is also possible to study the effect of quadrupole field imperfections in circular accelerators by applying a periodic perturbation to the pulse height as shown in Fig. 3. Needless to say, arbitrary lattice structures more complicated than these examples can be simulated as long as they are periodic.

For a systematic exploration of the space-charge effects, the plasma density must be controllable. In other words, it is desirable to provide some mechanism that enables us to adjust the *tune depression* η while keeping the waveform of the voltage function $V(z)$, we must change the *emittance* of the plasma that is invariant in a conservative dynamical system. The best way to survey a wide range of η is probably the introduction of some dissipative force for a direct control of the plasma temperature. For this purpose, the *laser cooling* method can be employed [11]. Since the mass m of confined particles is nothing but a parameter here, we can choose any ion species if it is coolable. For instance, $^9Be^+$ and $^{24}Mg^+$ are good candidates as they have a closed two-level transition; a single laser suffices for cooling this type of ions. In order to supply a gas of particular ions, an atomic oven is set beside the electrodes. Neutral atoms from an oven are ionized in the trapping region by thermal electrons from an electron gun. It is also possible to send ions, via a low-voltage acceleration gap, from a compact ion source attached to the trap. We should, however, bear in mind that the rms radius of the plasma must be modest since the spot size of a laser is usually not so large. R of less than several centimeters is thus preferable. The Doppler limit of laser cooling is in the mK level or even less, which suggests that we can cover the full range of tune depression from $\eta=1$ (high-temperature limit) to $\eta=0$ (low-temperature limit). Moreover, it is possible to directly and non-destructively measure the plasma motion at an extremely high resolution by detecting photons spontaneously emitted from excited ions. Amplifying the fluorescence by a photomultiplier and then catching it with a CCD camera, we can even image the real-time evolution of a tiny portion of the plasma. The number of trapped particles is not an essential matter at all. This is particularly important when we have to observe the behavior of such a low-density fraction as halos.

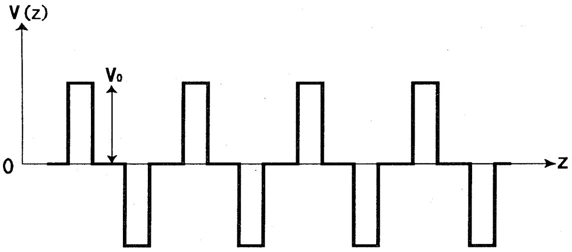

Fig.2 An example of $V(z)$ corresponding to a perfect FODO cell

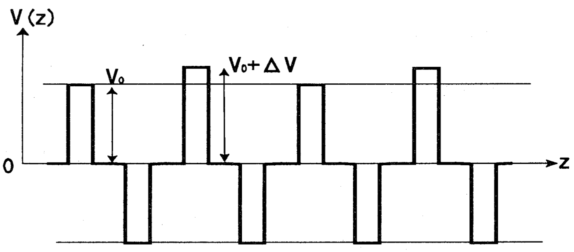

Fig. 3 An example of $V(z)$ corresponding to a FODO cell with field imperfection

4 SUMMARY

In this paper, we have pointed out an important analogy between charged-particle beams periodically focused by linear external fields and single-species plasmas confined in a linear Paul trap. As demonstrated in Section 2, the equation of plasma motion in the trap system is physically equivalent to that governing the collective beam motion. This fact offers the unique possibility of using a compact trap for the systematic experimental studies of space-charge-dominated beams. Considering the technical difficulties and various noise sources accompanied by accelerator-based experiments, this new approach will provide better information on the nonlinear behaviors of charged-particle beams.

REFERENCES

[1] R. W. Müller, in: G. Velarde, Y. Ronen, J. M. Martinez-Val (Eds.), Nuclear Fusion by Inertial Confinement: A comprehensive Treatise, CRC, Boca Raton, FL, 1993, p. 437.

[2] M. Reiser, T. Godlove, R. Bangerter (Eds.), Hearvy Ion Inertial Fusion, AIP Conference No/ 152, AIP, New York, 1986.

[3] R. A. Jameson, in: J. S. Wurtele (Ed.), Advanced Accelerator Concepts, AIP Conference Proceedings No. 279, AIP, New York, 1993.

[4] R. L. Gluckstern, in: P. M. Lapostolle, A. Septier (Eds.), Linear Accelerators, North-Holland, Amsterdam, 1967, p.827.

[5] F. J. Sacherer, IEEE Trans. Nucl. Sci. NS-18 (1971) 1101.

[6] T. P. Wangler, K. R. Grandall, R. S. Mills, M. Reiser, IEEE Trans. Nucl. Sci.NS-32 (1985) 2196.

[7] R. A. Jameson, IEEE Trans. Nucl. Sci. NS-28 (1981) 2408.

[8] I. Hofmann, L. J. Laslett, L. Smith, I. Haber, Part. Accel. 13 (1983) 145.

[9] M. Reiser, Theory and Design of Charged Particle Beams, Wiley, New York, 1994 and references therein.

[10] H. Okamoto, Hiroshima University Preprint HUBP-01/98 (1998); H. Okamoto and H. Tanaka, Nucl. Instrum. Meth. A437 (1999) 178.

[11] D. J. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. 20 (1975) 637; T. Hänsch and A. Schawlow, Opt. Commun. 13 (1975) 68.