DEVELOPMENT OF A TRACKING AND ANALYSIS CODE FOR BEAM DYNAMICS IN SPRING-8

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Abstract

A tracking and analysis code for baem dynamics has been developed in SPring-8. Exact Hamiltonian, not linearized, is used to derive equations of motion in the code. The equations of motion include radiation losses, if specified. These equations of motion are solved by symplectic integration method for basic devices such as drift space, bending, quadrupole and sextupole magnets, though filed errors and steering magnets are treated as kicks and RF cavities are also treated as thin elements. The code has been extended from four dimensions to six dimensions and effective analytical tools have been developed in this code system.

1 INTRODUCTION

Nowadays, one pursues ultimate performance of an accelerator, for example, high current of ampere order and sub-nmrad beam emittance. It is natural that one should design or modify an accelerator carefully and also optimize or re-optimize its parameters precisely. To this end, one needs a computer code which can exactly simulate nonlinear particle motion including direct and indirect particle-particle interaction.

We started to improve RACETRACK [1] which is a kick code on the basis of 4 by 4 linearized Hamiltonian in the beginning of SPring-8 project [2]. Many functions, i.e., a beam injection simulator, a closed orbit distortion (COD) correction package, a 4 by 4 optics and emittance calculation routine and so on were added step by step to meet our requirement. Toward the normal mode analysis on higher order nonlinear effects, well-known differential algebra algorithm [3] was introduced into the code to extract one turn map.

The code was modified drastically in the period from 1997 to 2000. We gave up the 4 by 4 linearized Hamiltonian, i.e., RACETRACK to precisely simulate beam dynamics in new optics with four magnet-free long straight sections [4,5]. Here, we adopted 6 by 6 exact Hamiltonian having full kinematics terms. This means that all components of an accelerator become nonlinear and we can not use a simple transfer matrix for any element. By expanding the code from 4 by 4 to 6 by 6 formalism, we can qualitatively estimate nonlinear particle motion with a large energy deviation and also particle motion under large chromaticity.

In parallel, we developed various analytical formulation on nonlinear beam motion [6,7,8] to analyze beam behavior in the real ring. Those were installed in our code as analytical tools. To make realistic simulation, it is also important to estimate real error distribution in the ring. From this point of view, a Model Calibration Method (MCM) package was developed [9, 10].

Recently, to estimate a momentum acceptance precisely and simulate top-up operation, we developed a synchrotron radiation handling package. This package has two options: the expected value option and the quantum photon option. We are now developing both a normal mode analysis package and a 6 by 6 optics and emittance calculation routine. We are also trying to include a short range wake-field effect for more precise simulation of real beam behavior.

2 CODE CONSTRUCTION

The code construction is shown in Fig.1. The ring parameter handler determines the lattice structure from element and structure data read from files specified by the job control data. Field error information is prepared by response analysis and taken into the code, if necessary [9,10]. Alignment errors can be included in the element data.

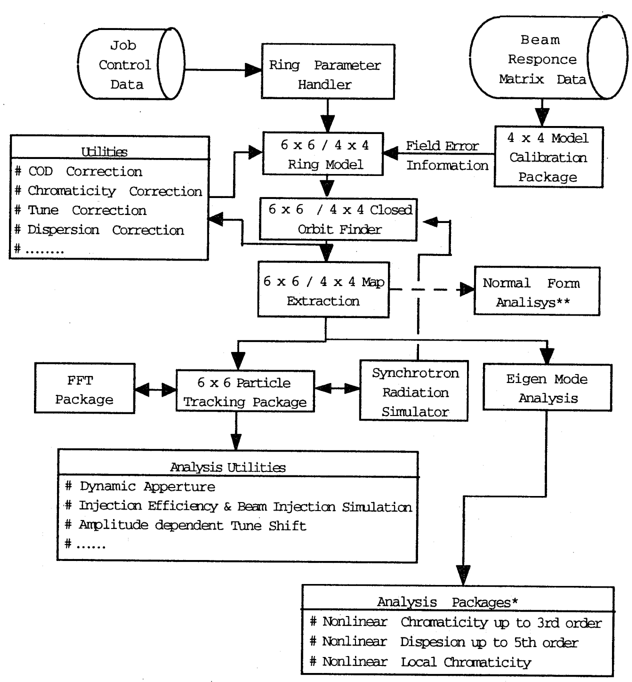

Figure 1: Code construction. A single asterisk indicates that the packages are in 4 by 4 formalism, and double asterisks indicate that the package is now under construction.

The closed orbit finder in 6 dimensional form determines the closed orbit with some correction utilities

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such as COD correction, chromaticity correction and so on, if necessary. When the lattice includes no cavities and the radiation option is not activated, this closed orbit finder uses 4 dimensional form. In 6 dimensional closed orbit search, RF frequency or momentum deviation $\delta p/p$ is searched dependending on the selected option. During closed orbit search, the code uses expected values of radiation energies. Then one-turn map in 6 dimensional form is determined.

After one turn map extraction, two functions are available in the code. One is the tracking and the other is the eigen mode analysis [6,7,8]. Normal form analysis package will be added in the code construction near future. Though the tracking package is in full 6 dimensional form, eigen mode analysis packages are not in 6 dimensional form yet.

In tracking package, the code has two radiation options, if radiation option is activated. One is the expected value option and the other is the quantum photon option. The expected value option uses expected value of radiation energy, while quantum photon option uses quantum photon energy determined by random number and energy spectrum of photons.

3 EQUATION OF MOTION

3.1 Hamiltonian

The Hamiltonian used in the code is given by

$$H = p_\sigma - h[\{(1+\delta)^2 - (p_x - qA_x/p_0)^2 - (p_y - qA_y/p_0)^2\}^{1/2} - qA_s/p_0], \quad (1)$$

where

$$\delta = (p - p_0)/p_0, \quad (2)$$

$$h = 1 + K_x x + K_y y, \quad (3)$$

$$p_\sigma = (E - E_0)/p_0 v_0 \quad (4)$$

and subscript 0 means the value corresponding to design ones, K_x and K_y are the curvature of the x- and y-direction, respectively. The Hamiltonian and canonical momenta p_x and p_y in eq.(1) are normalized by the product of design momentum and light velocity, p_0c. This Hamiltonian is basically the same as Ref. [6], but modified to include synchrotron motion term p_σ.

The equations of motion are as follows:

$$x' = \partial H/\partial p_x, \quad p_x' = -\partial H/\partial x + r_x, \quad (5)$$

$$y' = \partial H/\partial p_y, \quad p_y' = -\partial H/\partial y + r_y, \quad (6)$$

$$\sigma' = \partial H/\partial p_\sigma, \quad p_\sigma' = -\partial H/\partial \sigma + r_\sigma, \quad (7)$$

$$\sigma = s - v_0 t. \quad (8)$$

The symbol σ is the deviation of the longitudinal position from the bunch center. Radiation loss is considered as r_x, r_y and r_σ in eqs.(5) to (7). It is assumed that photons are emitted toward the direction of propagation. We have

$$A_x = A_y = 0 \quad (9)$$

for all the elements except insertion devices. Then the Hamiltonian becomes

$$\begin{aligned} H = p_\sigma &- (1 + K_x x + K_y y)[\{(1+\delta)^2 - p_x^2 - p_y^2\}^{1/2} \\ &- (1 + K_x x + K_y y)^2/2 + g_0(x^2 - y^2)/2 \\ &+ \lambda_0(x^3 - 3xy^2)/6 \\ &- (L/2\pi k)(q/p_0c)V\{\cos(2\pi k\sigma/L + \Delta\phi) \\ &+ \sigma\sin\phi_0\}]. \end{aligned} \quad (10)$$

The last term of eq. (10) is for the cavity, and $\phi_0 = 0$, if radiation option is activated. The symbol k is the harmonic number and L is the circumference of the ring. Here, multi-poles higher than sextupole are neglected, though they can be treated as kicks in the code. The equations of motion are derived from eqs. (5) to (7) and (10). The vector potentials used for the insertion device are basically the same as Ref. [11].

3.2 Symplectic Integrators

For Hamiltonian systems of the form

$$H = T(p) + V(q), \quad (11)$$

there exist explicit symplectic algorithms [12]. As the Hamiltonians for the quadrupole and sextupole magnets are given by the form of eq. (11), symplectic integrators for the quadrupole and sextupole magnets can be explicit integrators. We have adopted the fourth order integrator given as follows:

$$q_i = q_{i-1} + \tau c_i(\partial T/\partial p)_{p=p_{i-1}}, \quad (12)$$

$$p_i = p_{i-1} - \tau d_i(\partial V/\partial q)_{q=q_i}, \quad (13)$$

for i=1 to 4, where τ is the integration step width.

$$c_1 = c_4 = 1/[2(2 - 2^{1/3})],$$

$$c_2 = c_3 = (1 - 2^{1/3})/[2(2 - 2^{1/3})],$$

$$d_1 = d_3 = 1/(2 - 2^{1/3}),$$

$$d_2 = -2^{1/3}/(2 - 2^{1/3}), \; d_4 = 0. \quad (14)$$

Symplectic integrator for the drift space is the same as the transfer matrix except σ calculation, and the integration step width is taken as the length of the drift space, i.e. one step integration.

The Hamiltonian for the bending magnet is not given in the form of eq. (11), so the symplectic integrator for the bending magnet should be implicit, if generating function cannot be found. We have adopted the simplest first order implicit integrator given by

$$z' = z + \tau f((z + z')/2) \quad (15)$$

which is known as the implicit midpoint rule.

The explicit first order symplectic integrator for insertion devices can be derived by means of generating function [11].

Field errors, steering magnets and RF cavities are still treated as kicks. Thin lenses treated as kicks are essentially symplectic.

4 EXAMPLES

Some examples of the simulation are shown in this section. First, Fig.2 shows the experimental data and the results of horizontal chromaticity calculation. We find the thin lens approximation of sextupoles is not valid in the region where the absolute value of momentum deviation $\delta p/p$ is large. This shows that the thick magnet treatment of sextupole is essential in nonlinear chromatic behavior.

In Fig.2, the result of the 3rd order nonlinear chromaticity calculation is also shown. In the range data exist, results of exact 3rd order calculation and tracking with thick sextupoles agree with experimental data.

Secondly, tune shift was computed for BL19XU (25m undulator) which was installed in the long straight section of SPring-8 last year. Figure 3 shows the relation of tune shift and gap width calculated by the code. The agreement of the calculation with data is fairly well.

As an example of 6-dimensional calculation, Fig.4 shows ds(deviation from bunch center)-E(particle energy) phase diagrams for injection mode simulation. In Fig.4, initial σ_{ds}, standard deviation of the bunch length, is doubled to enhance the shape development of the phase diagrams.

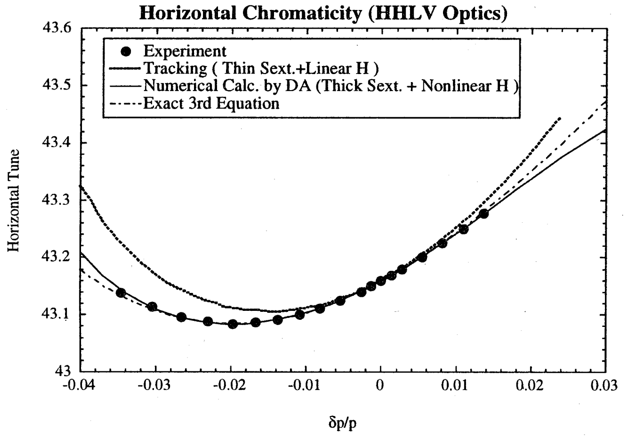

Figure 2: Typical horizontal chromaticity as a function of $\delta p/p$.

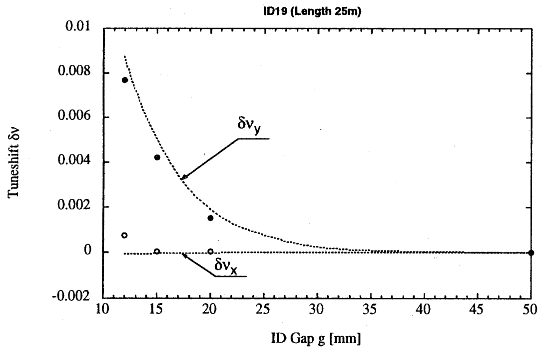

Figure 3: Tune shift by insertion devise. The open circles and solid circles are horizontal tune shift and vertical tune shift, respectively.

5 SUMMARY

We have developed a tracking and analysis code which uses equations of motion derived from exact Hamiltonian. The equations of motion are solved by symplectic integrators. Nonlinear effects such as dispersion and chromaticity can be explained by the code. The code developed simulates the nonlinear beam motion well in SPring-8 storage ring.

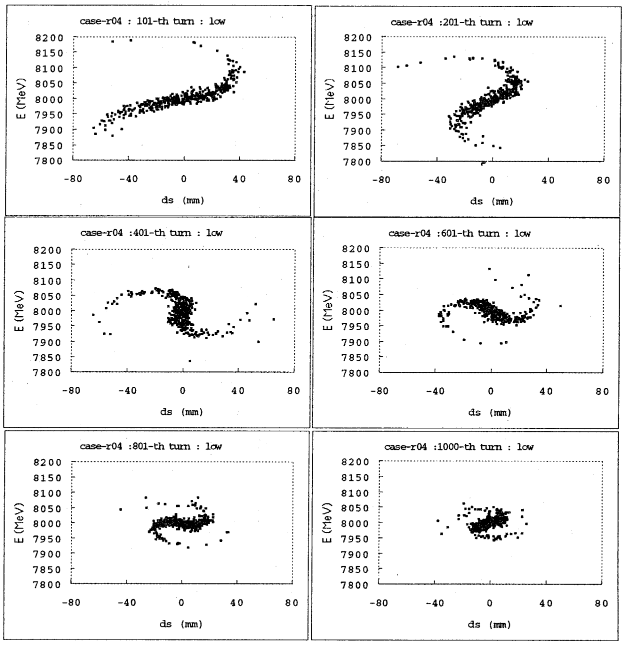

Figure 4: An example of ds-E phase diagram. From upper left to lower right, 101-th, 201-th to 1000-th turn after beam injection at each 200 turn step. Four hundreds macro-particles are injected.

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