

## Energy Spread Measurement of the SPring-8 Storage Ring with Chromatic Sideband Peak Height of Betatron Oscillation Spectrum

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### 1. Introduction

The energy spread of electron beams in storage rings are usually estimated from the measurement of bunch length, the transverse beam size at finite dispersion and, at low energy beam, the spectrum of light generated by transverse optical klystron which is a special type of insertion devices.

We propose a new method to measure the energy spread of the beam, using the relative height of chromatic sideband peaks in the frequency response of betatron motion[1]. The preliminary measurement of the energy spread of the electron beam in the SPring-8 storage ring was reported. In this paper, the definition of the chromaticity  $\xi$  is  $\Delta\nu\beta = \xi (\Delta E / E)$  where  $\Delta\nu\beta$  is the betatron tune shift and  $\Delta E/E$  is the relative energy shift of the beam. This definition is different from the Ref. [1] where chromaticity is a normalized value;  $\xi / \nu\beta$ .

### 2. Method

If we measure the frequency response of the betatron motion of a bunch in a storage ring which has finite chromaticity, we will observe several sideband peaks which apart from the main peak of betatron frequency with the distance  $n f_s$ , where  $n = 0, \pm 1, \pm 2, \dots$  and  $f_s$  is the synchrotron frequency. The typical data of the frequency response of the betatron motion of the SPring-8 storage ring is shown in Fig. 1.

Frequency Response of Betatron Motion

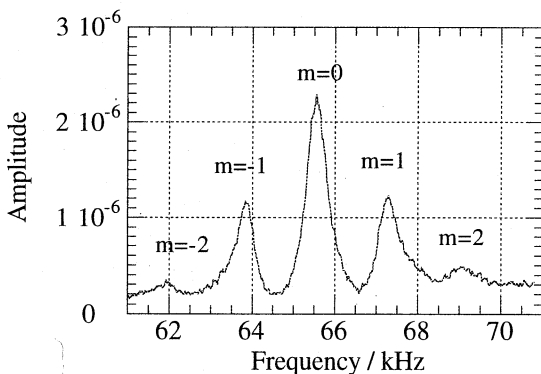


Fig. 1. Typical data of the frequency response of the betatron motion of the beam in the SPring-8 storage ring. The direction of betatron motion is vertical and the vertical chromaticity is 8.8. The height of the  $m=0$  peak is reduced to be a half of that at chromaticity 0 as shown in Fig. 2. The mode number

$m$  of each peak is also shown. This signal was averaged during 10 sweeps.

As shown in Ref. [1] for harmonic RF potential, the height of these chromatic sideband peaks depend only on the value  $y = \xi \sigma_\delta / \nu_s$  where  $\xi$  is the chromaticity,  $\sigma_\delta$  is the energy spread,  $\nu\beta$  and  $\nu_s$  is the betatron tune and synchrotron tune, respectively. The dependence of the peak heights on  $y$  are shown in Fig. 2. Fig. 3 shows the ratio of the peak height of the sideband to the peak height of the main peak ( $m=0$ ).

From the data of the height of sideband peaks, we have the values of  $y = \xi \sigma_\delta / \nu_s$  from Fig. 2 or Fig. 3. On the other hand, synchrotron tune  $\nu_s$  is obtained from the distance of sideband peaks each other or synchrotron frequency measurement and the chromaticity  $\xi$  is obtained from the dependence of betatron frequency on RF frequency using the relation  $\Delta\nu\beta = (\xi / \eta) (\Delta f_{RF} / f_{RF})$ , then we have  $\sigma_\delta$ .

From the Fig. 2 and Fig.3, the sensitivity is high at  $y \sim 1$  for  $m=\pm 1$  peaks and  $y \sim 2$  for  $m=\pm 2$  peaks.

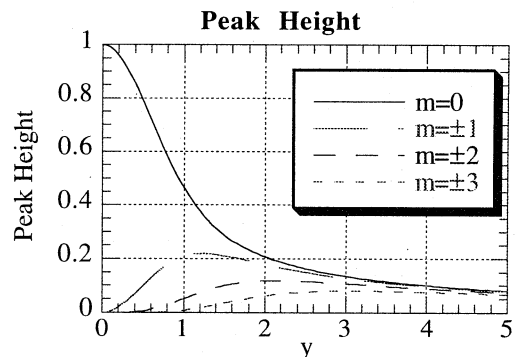


Fig. 2. Calculated relative peak height for  $m=0, \pm 1, \pm 2, \pm 3$  based on Ref.[1].  $y = \xi \sigma_\delta / \nu_s$

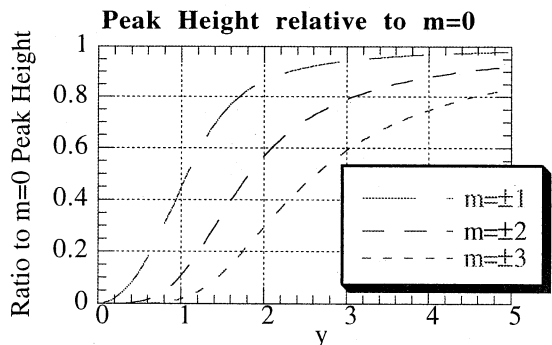


Fig. 3. Calculated ratio of peak height of  $m=\pm 1, \pm 2, \pm 3$  to the amplitude of the peak of  $m=0$ .  $y = \xi \sigma_\delta / \nu_s$ .

### 3. Experiment

The setup of the measurement is shown in Fig. 4 and is usually used as a tune monitoring system.

The signal to excite the betatron motion is generated by a tracking generator in the spectrum analyzer and is fed to the strip-line. The tracking generator generates a signal of the frequency tuned to the sweeping frequency of the spectrum analyzer hence we can obtain frequency response of the betatron motion of the beam.

The signal of the transverse motion of the beam was detected by a button-type electrode attached to the beam pipe of the ring and measured by the spectrum analyzer. The center frequency of the spectrum analyzer was tuned to the betatron frequency ( $m=0$  peak) and the span of the frequency was set to be 10kHz which is wider enough to cover sideband peaks of  $n = \pm 1, \pm 2$  at nominal value of the synchrotron frequency;  $\sim 2$ kHz.

The chromaticity was varied by changing the strength of sextupole magnets of the ring and measured by taking the dependence of the betatron tune on the RF frequency using the relation

$$\Delta\nu\beta = (\xi/\eta) (\Delta f_{RF}/f_{RF})$$

where  $\eta$  is the momentum compaction factor and was assumed to be  $1.46 \times 10^{-4}$ .

Spectrum Analyzer

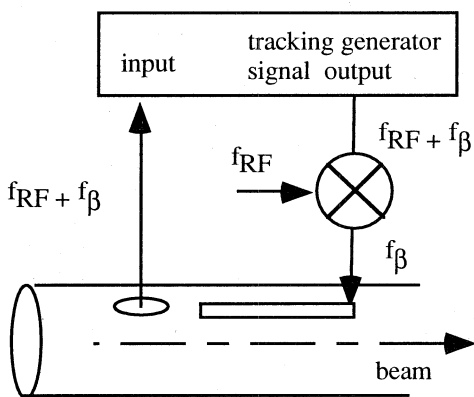


Fig. 4. Setup for measurement of frequency response of betatron motion of the beam in the storage ring. A spectrum analyzer generated the signal of frequency  $f=f_{RF} + f_{\beta}$ , where  $f_{RF}$  is RF frequency and  $f_{\beta}$  is the betatron frequency, and this signal was down-converted by a double-balanced mixer to the frequency  $f_{\beta}$  and fed to a strip-line electrode to shake the beam. The transverse motion of the beam excited by this signal is detected by a button-type pick-up electrode at frequency  $f=f_{RF} + f_{\beta}$ .

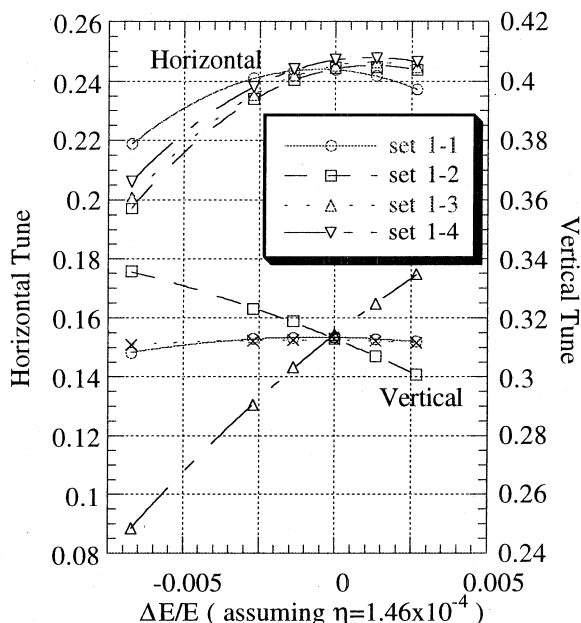


Fig. 5 Energy dependence of the betatron tune ( $m=0$ ) assuming the momentum compaction factor is  $1.46 \times 10^{-4}$ . Four lines are shown for different sets of sextupole magnet strength.

At the measurement of the synchrotron frequency, the synchrotron motion was excited by adding phase modulation to RF acceleration voltage and the frequency response of the phase oscillation amplitude of the beam was measured.

The dependence of the betatron frequency ( $m=0$ ) on the energy shift controlled by the RF frequency is shown in Fig. 5. The chromaticity is obtained by  $\xi = d\nu\beta/d(\Delta E/E)$ . We assume the momentum compaction factor  $\eta$  to be the design value;  $1.46 \times 10^{-4}$  to convert the RF frequency shift to the energy shift using the relation  $\Delta E/E = 1/\eta (\Delta f_{RF}/f_{RF})$

### 4. Results

The energy spread obtained from the relative peak height of  $m = \pm 1, \pm 2$  to the peak height of  $m=0$  using Fig. 3 are shown in Fig 6, 7. The energy spread obtained from peak height dependence of  $m=0$  peak on chromaticity using Fig. 2 is shown in Fig. 8. The error caused by reading error from the data which are recorded as video images of the spectrum analyzer display, is shown in Fig. 9. This figure shows that the error is lowest at the chromaticity 4 to 8. The measured value of the synchrotron tune  $\nu_s$  is 0.00778 at these measurement.

These result shows that the energy spread of the beam is  $1 \times 10^{-3}$  which is as expected from the magnetic field measurement of the dipole magnets[2].

**Energy Spread (from Horizontal data)**

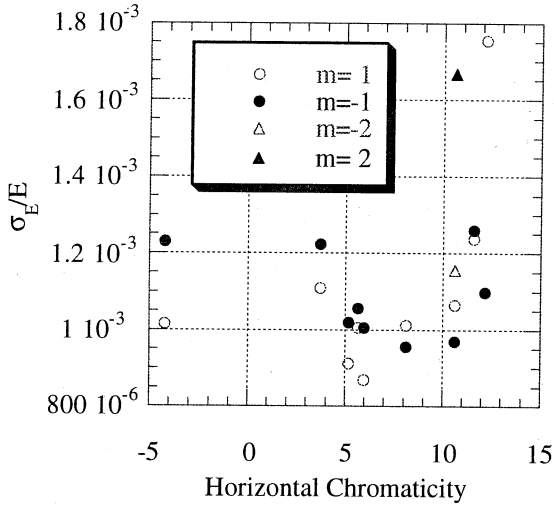


Fig. 6. The energy spread obtained from the ratio of chromatic sideband peak height relative to  $m=0$  main peak for horizontal betatron motion. From Fig. 9, the error is smallest at chromaticity  $\sim 4-8$  for  $m=\pm 1$  peaks and  $\sim 8$  for  $m=\pm 2$  peaks.

**Energy Spread (from Vertical data)**

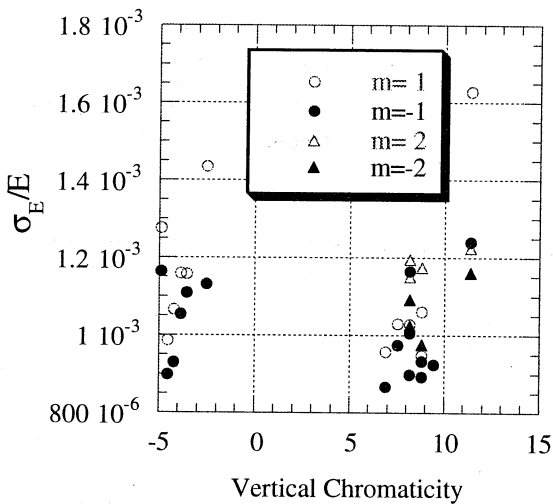


Fig. 7. The same as Fig. 6 but from vertical data.

**Betatron Amplitude ( $m=0$ ) vs Chromaticity**

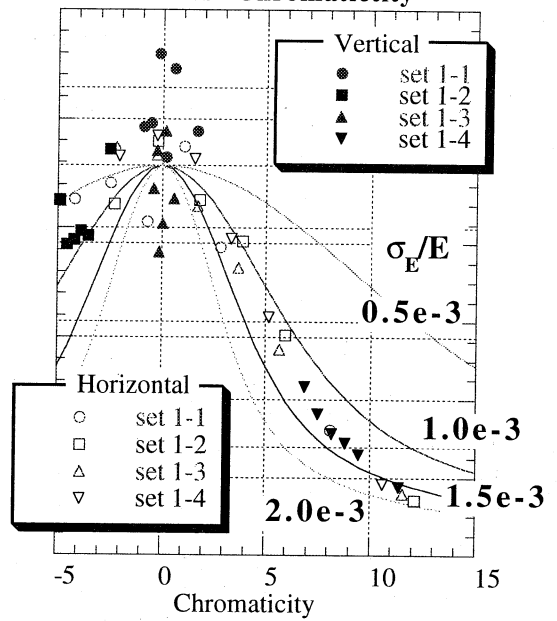
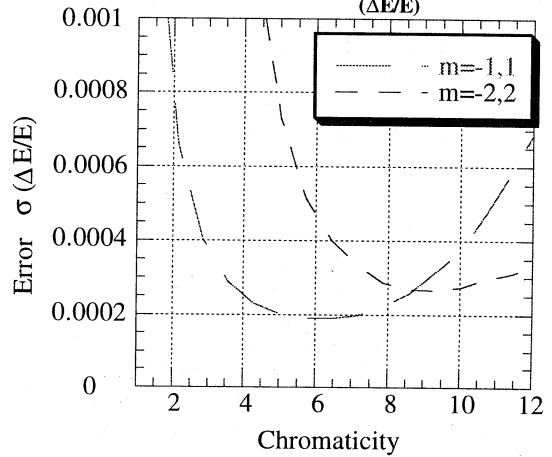


Fig. 8. Energy spread measured by the peak height of main peak  $m=0$  for different set of sextupole field strength. The lines in the figure are expected value of each energy spread which shown in the figure. This results show the energy spread is between  $1 \times 10^{-3}$  and  $1.5 \times 10^{-3}$ . Peak value at chromaticity  $=0$  is the average of the measured values.

**Error  $\sigma_{(\Delta E/E)}$**



(for  $f = 1634\text{Hz}$ ,  $\sigma_E/E = 1.1e-3$ )

Fig. 9. Estimated error of the measured energy spread using the ratio of peak height, caused by reading error from data as video images of the spectrum analyzer display recorded as video images. This figure shows that the error is smallest at chromaticity 4 to 8.

**References**

- [1] T. Nakamura, "Excitation of Betatron Oscillation under Finite Chromaticity", this proceedings.qq
- [2] K. Tsumaki, SPring-8 Annual Report 1998.