Modeling of a super-conducting cavity de-tuned by the Lorentz pressure.

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Abstract

A dynamic analysis model for super conducting cavities under the pulse operation is presented. The model treats transient behavior of the amplitude and the phase of the accelerating field excited by both a generator and a beam, hence the cavity is de-tuned due to the wall deformation caused by the strong pressure of the field. Simulations for a 600MHz single cell cavity (E=16MeV/m) were done to demonstrate field response for the 4ms pulse operation. Performances of the feed back control of the accelerating field were demonstrated as well, which show a possibility of stabilizing the field less than 1% and 1deg with the sampling duration of 100kHz.

1 Introduction

A pulse operation of a super conducting accelerating cavity results in the cavity deformation accompanied with de-tuning of the resonant frequency [1]. In the Joint project [2] of Japan Hadron collider facility (JHF) of KEK and Neutron Science Project (NSP) of Japan Atomic Energy Research Institute (JAERI), the super conducting cavities play the roll of the accelerating structure from 400MeV to 600MeV. In this design, accelerating field stability is the one of the great concerns because of the pulse operation and the intense field level (up to 20MV/m).

As the cavity deformation causes the resonant frequency shift of the RF field, the response of the field, which is the source of the pressure on the cavity surface, and the response of the deformation should be treated consistently. The analysis tool for this transient behavior of the accelerating field coupled with mechanical vibration has much importance for the system design of the linac[3]. In this paper, equation sets for the cavity modeling are described and some simulation results for the 600 MHz cavity design are presented.

2 Modeling of the cavity

2.1 Electrical response

Figure 1 shows an equivalent circuit of a cavity. Ig is the current of a generator, Ib is the beam current, Vc is the voltage across the accelerating gap, and Rs is the shunt impedance of the cavity. The cavity is excited to a generator with frequency $f = \omega / 2\pi$ with a coupling coefficient of β. The amplitude and the phase of the cavity voltage are presented by following first order differential equations.

$$\tilde{V}_c = V_c e^{i(\omega t + \phi)} \tag{1}$$

$$\tilde{V}_{gr} = V_{gr} e^{i(\omega t + \phi_g)} \tag{2}$$

$$\tilde{V}_{br} = V_{br} e^{i(\omega t + \phi_b)} \tag{3}$$

$$\tau \dot{V}_c = V_{gr}\cos(\phi_c - \phi_g) - V_{br}\cos(\phi_c - \phi_b) - V_c \tag{4}$$

$$\tau \dot{\phi}_c = -\frac{V_{gr}}{V_c}\sin(\phi_c - \phi_g) + \frac{V_{br}}{V_c}\sin(\phi_c - \phi_b) - y \tag{5}$$

$$Q_L = \frac{Q_0}{1+\beta}, \tag{6}$$

where, Vgr, Vbr and y are defined as,

$$\tilde{V}_{gr} = \frac{R_S}{(1+\beta)}\tilde{I}_g, \tag{7}$$

$$\tilde{V}_{br} = \frac{R_S}{(1+\beta)}\tilde{I}_b, \tag{8}$$

$$y = -\tan\Psi = 2Q_L\frac{f - f_r}{f_r} = -2Q_L\frac{\Delta f}{f_r}. \tag{9}$$

f_r is the resonant frequency of the cavity, which is de-tuned from generator frequency with angle Ψ.

2.2 Mechanical response

Displacement vector {u} of the cavity wall from design position is expressed by an expansion of eigen vectors $\{a_k\}$ of mechanical oscillations.

$$\{u\} = \sum_{k}^{2n}\{a_k\}\xi_k\cos(\omega_k t + \varphi_k) \tag{10}$$

ξ_k and ω_k are the amplitude and the angular frequency of the k-th mode oscillation. The oscillations of each mode are presented by a following pendulum equation.

$$m'_k\ddot{\xi}_k + c'_k\dot{\xi}_k + k'_k\xi_k = \{a_k\}^T\{F\} \quad \{k = 1,2,...n\} \tag{11}$$

m'_k, c'_k, k'_k are so called generalized mass, damping factor and elastic constant of the k-th mode. Lorentz force distribution on the cavity wall {F} is the function of square of cavity voltage. Using the nominal voltage V_0 and the force $\{F_0\}$,

$$\{F\} = \left(\frac{V_C}{V_0}\right)^2\{F_0\} \tag{12}$$

Resonant frequency shift due to the k-th mode oscillation is given by

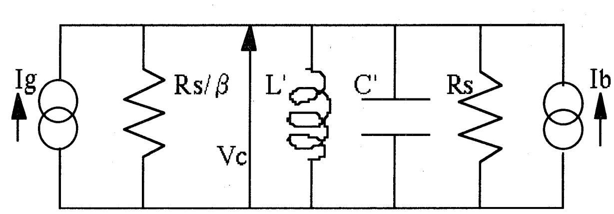

Fig. 1 Equivalent circuit of the cavity.

$$\Delta f_k = \frac{\partial f}{\partial\{u\}}\{a_k\}\xi_k = \frac{\partial f}{\partial \xi_k}\xi_k \quad . \tag{13}$$

Thus, as a total frequency shift is,

$$\Delta f = \sum_k^{2n} \frac{\delta f}{\delta \xi_k}\xi_k \quad . \tag{14}$$

Solving equations from (1) to (14) with time step calculation, transient behavior of the cavity voltage, phase, and the mechanical oscillation can be obtained consistently.

Lorentz force $\{F_0\}$ at nominal voltage and frequency shift sensitivity due to the wall deformation $\partial f/\partial\{u\}$ can be calculated using the simulation code Superfish. Mechanical resonant frequency $2\pi f_m = \sqrt{k/m}$ and coefficient for each mode m'k, c'k can be estimated using the simulation codes like ABQCUS, or more directly, can be measured in modal test of existing cavity itself.

The cavity model was programmed using "MATLAB/ Simulink"[4]. Simulink is a system for nonlinear simulation that combines a block diagram interface on the technical computing language MATLAB. One can easily add the cavity model with external feed back loop or other virtual devices like monitor of the field in the cavity. Simulations described in following section were performed on Simulink.

3 Simulation results for a 600MHz single cell cavity

3.1 Cavity parameters

A 600MHz cavity was chosen to demonstrate the simulation of field response for a pulse operation. Characteristic parameters for the 600MHz design used in the simulations are listed in table 1.

Accelerating field of 16MV/m corresponds to the total voltage for a 5-cell cavity of 3.3MV. Beam current of 20mA is equivalent to peak current of 33mA with duty factor of 60%. The cavity is matched to this beam current with the coupling coefficient of 14698 and the time constant of the field becomes 0.36ms.

Cross sectional view of the 5 cell cavity was shown in fig.2. The cavity is the bell shape one made of Niobium, with beam aperture of 150mm in diameter, and cell lengths of from 145.8mm to 150.8mm. The wall thickness of the cavity assumed to be 3mm.

Table 3 is the characteristics of mechanical oscillation modes calculated by ABAQUS, with the Superfish results of Lorentz force frequency shift sensitivities converted to the modal components using equation (11) and (13). Most of the frequency shift components under the condition of the statistic nominal field come from the mechanical modes less than the third mode. These modes play important roll in the transient response for the pulse excitation as well, because their frequencies are near by the frequency range of the filling time or the RF pulse width (around 1kHz).

3.2 Response for the pulse operation

Response of a single RF pulse was shown in fig.3a for the amplitude and fig.3b for the phase. Beam pulse arrives at 1ms after the generator pulse. Phase of the generator was programmed to jump as much as 30 degree between before and after the beam pulse arrival in order to compensate the beam loading effect.

Table 1
Parameters for the simulation

Operation frequency	600MHz
Proton velocity v/c	0.604
Unloaded Q value	1.1e10
Shunt impedance	1.4e12 ohm
Nominal accelerating field	16MV/m
RF pulse width	4ms
Beam pulse width	3ms
Repetition rate	50Hz
Average beam current	20mA
Accelerating phase	-30deg
Cavity de-tuning	+30deg
Voltage stability requirement	<+/- 1%
Phase stability requirement	< +/- 1deg

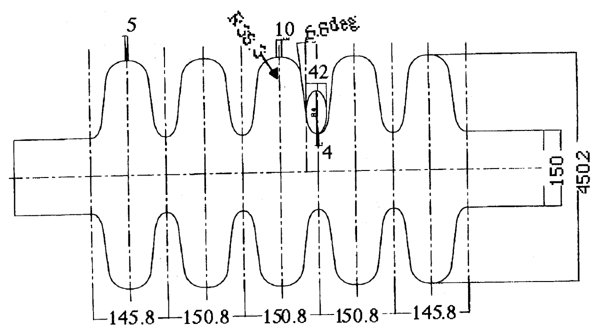

Fig. 2 Cross sectional view of the 600MHz 5cell cavity for beam velocity (v/c) of 0.604.

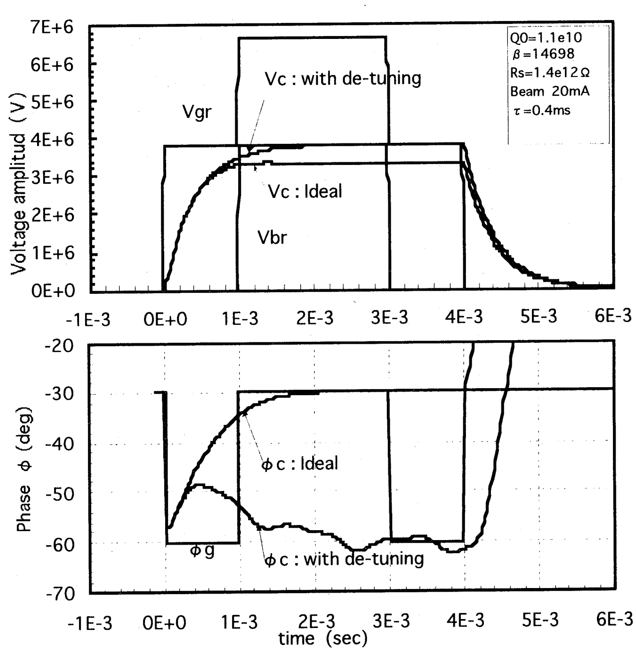

Fig. 3 Response of the cavity voltage for the pulse operation. (a):amplitude, (b) phase.

Table 2
Characteristics of the mechanical oscillation modes of the cavity. (f=600MHz, v/c=0.604, thickness=3mm)

Mode number	1	2	3	4	5	6	7	8	9	10
Resonance (Hz)	758.0	1594.5	2452.9	3441.5	4177.6	5105.0	6776.2	8814.9	10954.	11407
Gen. Mass (ton)	1.34E-3	1.53E-3	1.71E-3	1.55E-3	3.12E-3	1.25E-3	1.30E-3	1.37E-3	9.53E-4	1.12E-3
Force (kN)	3.29E-3	-7.28E-3	-6.18E-3	9.81E-4	3.03E-3	-3.81E-3	-2.54E-3	-7.46E-4	-6.61E-3	-1.07E-2
Amplitude(mm)	1.08E-4	-4.72E-5	-1.52E-5	1.35E-6	1.41E-6	-2.96E-6	-1.08E-6	-1.78E-6	-1.46E-6	-1.86E-6
Sensitivity(Hz/mm)	-4.36E+5	2.15E+6	1.82E+6	-6.45E+4	-8.22E+5	1.13E+6	7.13E+5	2.10E5	1.81E+6	2.90E+6
Frequency shift at nominal field (Hz)	-47.3	-101.2	-27.7	-0.0872	-1.16	-3.34	-0.768	-0.0374	-0.0374	-5.39

As to the simulation result, The amplitude of the induced cavity voltage Vc goes over more than 20% compared to the ideal response without de-tuning effects of Lorentz force. The phase is not correctly controlled in this condition as well. The response of the resonant frequency shift is shown in fig.4.

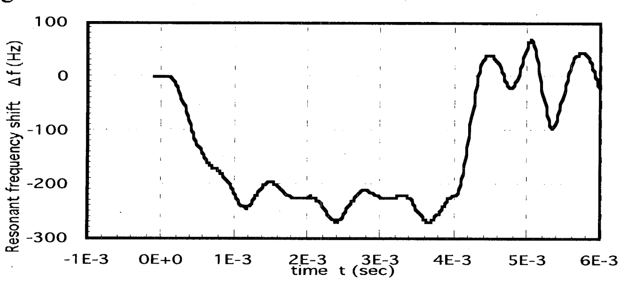

Fig. 4 Response of the resonant frequency of the cavity for the pulse operation.

3.3 Feed back control of the field

To stabilize the field, a simple feed back of the amplitude and phase was demonstrated. Block diagram of the system was shown in fig 5. Pick up signal from the cavity field was compared with the reference curve of the ideal response, then feed backed to the generator output with a sampling rate of 100kHz.

The results were shown in fig. 6a and fig.6b. Both the amplitude and the phase response were improved significantly. The simulation results suggest a possibility of the stabilization of the amplitude and the phase less than the required levels, 1% and 1degree respectively. More detail

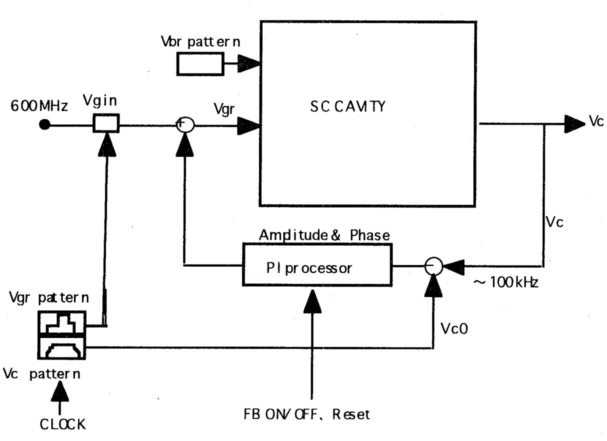

Fig. 5 Block diagram of the feed back system.

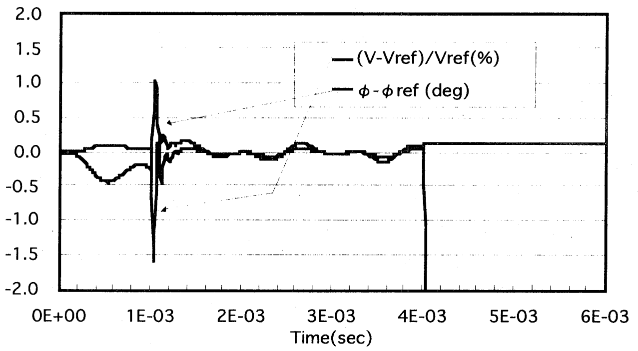

Fig. 6 Stability of the cavity voltage with the feed back gains of P=10, I=1

discussions for feed back control are presented in reference [5].

4 Conclusion

In order to avoid the problems related to the Lorentz force in the pulse operation, more realistic modeling of the feed back control system or improvement of the cavity stiffness should be studied in detail. The dynamic analysis tool for the electrical and mechanical response of the super conducting cavity presented here will be quite useful for further study on the field stabilization.

Acknowledgement

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