RF signal Optimization for Beamloaded Accelerator Control

M. A. Chernogubovsky
Department of Reactor Engineering, Japan Atomic Energy Research Institute,
Tokai-mura, Naka-gun, Ibaraki 319-11, JAPAN

Abstract

Transient beam excitation analysis for resonant accelerator allows to determine the control signal for minimization of required RF source power with simplification of the signal implementation and with RF system bandwidth minimization at prescribed errors in the field magnitude. Initial results on the accelerating channel optimization are obtained also.

1 Introduction

Effective RF control [1] in the intense beam resonant accelerator for single mode operation is realized at directional selective coupling, therewith RF source signal $S_s(t) = S_-(t) + S(t)$, where $S_-(t)$ sets up the required single harmonic $A\cos(\omega_0 t + \varphi)$ field and $S(t)$ compensates the beam excitation from $t = 0$ instant in $\omega \in [\omega_0 - y_0, \omega_0 + y_0]$ frequency band, [2]. RF system tuning characteristics and S signal are defined by the particles total energy derivative $F(t)$ (Eq.(6) in [2]).

Assuming the required signal $S_s(t)$ is synthesized and the control system operates ideal, the particles energy variation will be $T = 2\pi/\omega_0$ periodic, $\sum_a \mathcal{E}_a(t) = \sum_{n=1}^{N} \mathcal{E}_0(t-(n-1)T)$, where NT is the beam duration ($N \gg 1$), $\mathcal{E}_0(t)$ is the summary energy variation of the particles that enter the resonator at $t \in [0, T]$, the last particle from these leaves the resonator at $t = KT + \zeta$ ($K \ll N,\ 0 \leq \zeta < T$), and $\mathcal{E}_0(t)|_{t \geq KT+\zeta} = \text{const}(t) = \mathcal{E}_{out}$. Thus,

$$F(\omega) = \frac{-\omega_r D(\omega)}{W_1} \cdot \frac{\sin(\frac{\pi\omega}{\omega_0}N)}{\sin(\frac{\pi\omega}{\omega_0})} e^{-i\frac{\pi\omega}{\omega_0}(N-1)}, \qquad (1)$$

where $D(t) = \frac{d\mathcal{E}_0(t)}{dt}$ and also $D(t)|_{t \notin (0, KT+\zeta)} = 0$, ω_r, W_1 - resonant frequency and norm, and $\Lambda = \int_{-y_0}^{y_0} |F(\omega)|^2 d\omega / A^2$ value, preassigned at S energy E_0 minimization in [2], with any finite precision for $y_0 < \omega_0/2$ is $\Lambda = \mathcal{E}_{out}^2 N \omega_0 \omega_r^2 A^{-2} W_1^{-2}$. Basic specifications of the accelerated beam for prescribed resonator are included indeed, but [2] optimization is restricted by the phase θ fixation of the signal carrier $\propto \cos(\omega_0 t + \theta)$ from the E_0 minimum minimorum estimation. Complete solution is obtained at $S = (C_{0e}(\omega) + C_{0o}(\omega))\exp(i(\theta + \breve{o}(\omega) + \breve{e}(\omega))$ consideration in the general form of the even $C_{0e}(\omega), \breve{e}(\omega)$ and the odd $C_{0o}(\omega), \breve{o}(\omega)$ parts (for definiteness $\breve{e}(0) = 0$), Eq.(11) in [2] takes the form $\int_{-y_0}^{y_0} \chi \cdot (\overset{*}{\chi} + \chi\cos(2\varphi - 2\theta - 2\breve{e}(\omega)))d\omega = 2\Lambda$, where $\chi = C_{0e}(\omega) + iC_{0o}(\omega)$, $(^*)$ – complex conjugate, and by applying Cauchy-Schwarz inequality the amplitude modulated signal optimum is proved for any θ with global minimum of required energy

$$\mathsf{E}_0\pi = \Lambda/\cos^2(\varphi - \theta). \qquad (2)$$

Since the second factor in (1) is ω_0 periodic, the form

$$D = M(t)\cos(\omega_0 t + \varphi)\cos(\omega_0 t + \theta) + L(t)\cos(\omega_0 t + \varphi), \quad (3)$$

where $M(\omega)$ is not wider than $\omega \in (-\omega_0, +\omega_0)$, $L(\omega)$ is not wider than $(y_0 - \omega_0, \omega_0 - y_0)$ and also $M(t) = L(t) = 0$ at $t \notin (0, KT + \zeta)$, presents all possible solutions of the optimization conditions (14) in [2]. Thus, the beam excitation signal in the optimized accelerator is

$$V(t) = \frac{\cos(\omega_0 t + \theta)\sum_{k=0}^{N-1} M(t-kT) + \sum_{k=0}^{N-1} L(t-kT)}{-AW_1\omega_r^{-1}}; \quad (4)$$

the output energy is defined by $M(\omega)$ function only, since $\mathbf{F}\{L(t)\cos(\omega_0 t + \varphi)\}$ does not contain a constant component:

$$\mathcal{E}_{out} = D(0) = M(0)\cos(\varphi - \theta)/2. \qquad (5)$$

2 RF signal determination

The first term in exp.(4) presents the (ω_0) band excitation that must be compensated by S signal. As far as $M(t)|_{t \notin (0, KT+\zeta)} = 0$, the envelope $M_\Sigma(t) = \sum_{k=0}^{N-1} M(t-kT)$ is periodic: $M_\Sigma(t) = M_\Sigma(t+T)$ for $t \in [\tau_v, (N-1)T]$, $\tau_v = (K-1)T + \zeta$. For sufficiently large t this term time-dependence could be obtained from the infinite periodic excitation in (ω_0) band, that leaves behind a single harmonic from the line spectrum and the envelope is constant; without loss of generality the form

$$M(t) = f(t) - f(t-T), \qquad (6)$$

$$f(t)|_{t \leq 0} = 0,\ f(t)|_{t \geq \tau_v} = \text{const} = f_s,\ f'(\tau_v) = 0, \quad (7)$$

satisfies this condition,

$$M_\Sigma(t) = f(t) - f(t - NT), \qquad (8)$$

exp.(6) integration for Eq.(5) yields the stationary level

$$f_s = 2\mathcal{E}_{out}/(T\cos(\varphi - \theta)). \qquad (9)$$

Since the envelope energy in $[-y_0, +y_0]$ band is determined for any $M_\Sigma(t)$ by Eq.(2),

$$\frac{1}{4}\int\limits_{-y_0}^{y_0} |\mathbf{F}\{f(t) - f(t - NT)\}|^2 \, d\omega = \frac{\mathcal{E}_{out}^2 N\omega_0}{\cos^2(\varphi - \theta)}, \quad (10)$$

the most complete compensation is obtained for envelope with the minimal spectrum width[1]; the closely approximating acceptable form[2] is

$$f(t) = f_s \cdot \begin{bmatrix} \dfrac{e^{-\frac{(x-u)^2}{a^2}} - e^{-\frac{u^2}{a^2}}}{1 - e^{-\frac{u^2}{a^2}}}, & t \in [0, \tau_v] \\ 1, & t \in [\tau_v, \infty); \end{bmatrix} \quad (11)$$

where $x = t/T$, $u = \tau_v/T$; parameter a remains free, because the low-frequency filtering in Eq.(10) at $N \gg K$ eliminates a dependence on a, so that the equation yields only $N \gg 2K + \omega_0/(2\pi y_0)$ additional estimation for N lower bound.

To minimize the required bandwidth y_0 consider the errors of RF control system. The beam excitation spectrum is infinite in the any case (the finite time duration signal), and uncompensated outside of the $2y_0$ band signal (4) excites $e_{1un}(t)$ field (Eq.(3) in[2]), which maximal normalized amplitude estimation is

$$(|e_{1un}|/A) \le (|e_{1unM}|/A) = \frac{\int\limits_{y_0}^{gr} \frac{|M_{\Sigma 1}(\omega)|}{f_s} \left[\frac{1}{\sqrt{1+\xi_L^2(\omega+\omega_0)}} + \frac{1}{\sqrt{1+\xi_L^2(\omega_0-\omega)}} \right] d\omega}{2\pi(2+b)b^{-1}\cos(\varphi - \theta)}, \quad (12)$$

where $\xi_L(\omega) = \dfrac{\omega^2 - \omega_r^2}{\omega\omega_r} Q_L$, Q_L - loaded quality factor; the integrand function resonance filtration allows to bound the upper limit gr and to neglect the L functions sum contribution from exp.(4); b is the beam power to the cavity power losses ratio. Here $|M_{\Sigma 1}(\omega)| = |\mathbf{F}\{f(t)\}|$ is used instead of the general $|M_\Sigma(\omega)|$, because the constant component is excluded and the maximal $e_{1un}(t)$ value is attained at the commensurable with KT instant. This type error dependence is presented on Fig. 1 at concrete parameters (Q_r - selfquality factor, the frequency deviation with the loaded quality factor are determined by Eq.(8,9) in [2] for RF matching realization), but the similar dependence takes place in the general case also.

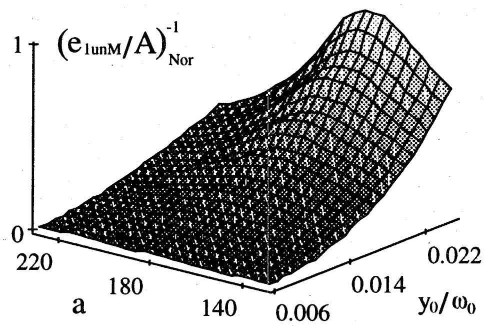

Fig. 1 Reciprocal off-band overshoot (normalized); $b = 1.1/1.68$, $u = 367, Q_r = 10^4$ values are taken from [5,6], $\cos(\varphi - \theta) = 0.5$.

The second type error is caused by not ideal compensation inside $2y_0$ band. For normal error difference between V and S, which power density spectrum is $w(\omega)$, the mean square field error is $\tilde{e}_{1a} = \sqrt{\frac{1}{\pi}\int\limits_{\omega_0-y_0}^{\omega_0+y_0} w(\omega) \left|\frac{Q_L}{\omega_r^2(1+i\xi_L(\omega))}\right|^2 d\omega}$. Since this error increases with the band extension while (12) error decreases, the optimal y_{0m} value in the meaning of the minimal total error can be presupposed, however, the normalized $\tilde{e}_{1a}$ value

$$\frac{\tilde{A}}{A} = \frac{1}{A}\sqrt{\frac{1}{\pi}\int\limits_{\omega_0-y_{0m}}^{\omega_0+y_{0m}} w(\omega) \left|\frac{Q_L}{\omega_r^2(1+i\xi_L(\omega))}\right|^2 d\omega} \quad (13)$$

must be prescribed as far as this error exists during all the accelerating time interval; therewith the autocontrol system precision Δ_a is specified by the normalized power error at the determined (Eq.(2)) S power,

$$\Delta_a = \frac{NT}{\mathrm{E}_0 \pi}\int\limits_{\omega_0-y_{0m}}^{\omega_0+y_{0m}} w(\omega) d\omega. \quad (14)$$

Assuming the autocontrol error to be white noise, Eq.(12),(13) immediately yield the equation for the extremal bandwidth values. Minimal bandwidths exist; overshoot dependence (Eq.(12) and Fig. 1) determines the range of a values that restricts the number of the extremals, and under the reasonable condition $(e_{1unM}/A) \ll \left(\tilde{A}/A\right)$ the optimum (a_{opt}, y_{mopt}) can be determined. The minimum dependencies for $e_{1unM} < 10^{-2} \cdot \tilde{A}$ at $y_0/\omega_0 < 0.025$ are presented in Fig. 2;

[1] The width can be defined by $\Omega^2 = \int_{-\infty}^{\infty} |M_\Sigma(\omega)|^2 \omega^2 d\omega$ value similarly to moment method [3,4] as far as the sharply defined $M_\Sigma(0)$ component is prescribed; and the function $G(\omega) = \mathbf{F}\left\{[\mathbf{1}(t+\tau_v) - \mathbf{1}(t-\tau_v)]\left(f(\tau_v - t) + f(\tau_v + t)\right) - f_s\right\}$, which time duration is evaluated by the accelerating periods number, gives the same value as $M_\Sigma(\omega)$ for any $f(t)$ that satisfies Eq.(7,8). Cauchy-Schwarz inequality consideration for $\sqrt{\int_{-\infty}^{\infty} t^2 G(t)^2 dt} \cdot \sqrt{\frac{\Omega^2}{2\pi}}$ with additional condition $f'(0) = 0$ yields $\Omega \ge \left(\pi \int_0^{\tau_v} f(t)^2 dt\right)^{\frac{1}{2}} / T_G$, where T_G – equivalent duration of $G(t)$. The last integral is limited (to be more precise, this energy in $[-y_0, +y_0]$ band is prescribed, but it can not be determined under $N \gg K$ condition), $T_G \le \tau_v$, and Ω minimum is achieved in the equality case, i.e., at $\frac{dG(t)}{dt} \propto tG(t)$ that is satisfied at $\tau_v \to \infty$ for Gauss time-dependence $G_\infty(t)$.

[2] Real $f(t)$ can not present $G_\infty(t)$ exactly, but under the use of this form the loss

$$\Omega/\Omega_{min} = \sqrt{\mathrm{erf}(\sqrt{2}\frac{u}{a}) - \frac{1}{\sqrt{2\pi}}\frac{u}{a}e^{-2(\frac{u}{a})^2}} / \left(1 - e^{-(\frac{u}{a})^2}\right),$$

erf - error function, is not very substantial for $u/a > 2$.

at the use of, e.g., the upper band ($a_{opt} \cong 184$), the loss is $\Omega/\Omega_{min} = 1.019$, so that any other possible solutions could give not more than 2% advantage in the bandwidth value.

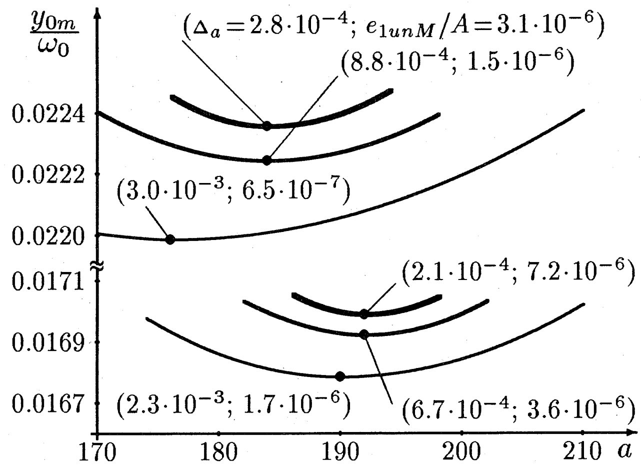

Fig. 2 Minimal bandwidths at $\tilde{A}/A = 10^{-3}$ and required control precision Δ_a, off-band overshoot e_{1unM}/A values in optimal points for nominal (—), doubled (—), and half (—) beam currents; $u = 367$, $b = 1.1/1.68, Q_r = 10^4$ values are taken from [5,6], $\cos(\varphi-\theta) = 0.5$.

Thus, the required signal in $\omega \in [\omega_0 - y_0, \omega_0 + y_0]$ is

$$S(\omega) = \frac{Ab\,\omega_r^2 \mathbf{F}\left\{\frac{f(t)-f(t-NT)}{f_s}\cos(\omega_0 t + \theta)\right\}}{Q_r \cos(\varphi-\theta)}, \quad (15)$$

where $f(t)$ is defined by Eq.(11,9) for prescribed $\mathcal{E}_{out}$, N, K, θ values; the optimized a_{opt}, bandwidth y_{mopt}, and required autocontrol precision are determined by Eq.(12,13) and Eq.(14) at prescribed random error and maximal tolerable overshoot error in the field magnitude.

3 Accelerating channel characteristics

As far as $M(t)$ is already defined, exp.(3) integration at separating out the terms without high-frequency oscillations gives the accelerated particles energy variation, averaged over the period

$$\mathcal{E}_{0A}(t) = \frac{\mathcal{E}_{out}}{1-e^{-\left(\frac{u}{a}\right)^2}} \cdot \begin{cases} \frac{a\sqrt{\pi}}{2}\left[\mathrm{erf}\left(\frac{x-u}{a}\right)+\mathrm{erf}\left(\frac{u}{a}\right)\right] - xe^{-\left(\frac{u}{a}\right)^2}, & x\in[0,1] \\ \frac{a\sqrt{\pi}}{2}\left[\mathrm{erf}\left(\frac{x-u}{a}\right)+\mathrm{erf}\left(\frac{x-u-1}{a}\right)\right] - e^{-\left(\frac{u}{a}\right)^2}, & x\in[1,u] \\ (x-u) - e^{-\left(\frac{u}{a}\right)^2} - \frac{a\sqrt{\pi}}{2}\mathrm{erf}\left(\frac{x-u-1}{a}\right), & x\in[u,u+1] \\ 1-e^{-\left(\frac{u}{a}\right)^2}, & x\in[u+1,\infty); \end{cases}$$

$\mathcal{E}_{0A}(t)$ laws are rather different for different a, Fig. 3. However, under the optimized a value the optimal RF field characteristics are obtained with actually insensitive to the beam current variations, Fig. 2, so that the channel with $a = a_{opt}$ is the best.

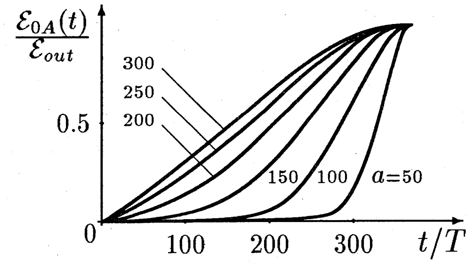

Fig. 3 Normalized energy for different a; $u = 367$.

4 Conclusion

The developed optimizations under effective RF control provide the following. (1) The beam excitation does not contain phase modulation. (2) The required RF source power is minimal at prescribed resonator. (3) RF source bandwidth is minimal for prescribed random error and maximal admissible overshoot error in the field magnitude. The optimization gives only a few percents disadvantage in reference to unrealizable minimal bandwidth limit. (4) RF signal implementation is the simple. (5) The optimized characteristics have only a weak dependence on the beam current deviations.

Concerning the beam dynamics, the (1) and (5) results allow to presuppose the minimal particle losses. The extremums are global (in the same meaning as indicated for (3) result), so that the use of the averaged energy variation law, which is maximally correlated with the optimized one will provide the better results if it can not be implemented exactly.

References

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