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Design Parameters of a Spiral Inflector for Axial Injection Project of the CYRIC Cyclotron

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Abstract

A derivation is given of the Root's solution for the beam orbit in the spiral inflector. On the basis of this solution a survey of parameters of the inflector was made, and appropriate design parameters were proposed considering the central-region geometry of the cyclotron.

1. Introduction

A preliminary numerical study of an axial injection scheme was made on the basis of an *off-axis* injection and a mirror inflector.[1] Since this scheme requires rather complex deflection electrodes within the central plug through the yoke, we abandoned this and turned to using a spiral inflector[2] together with an *on-axis* injection.

In the following we give a derivation of the Root's solution[3] and then apply this to our CYRIC cyclotron,[4] assuming an *on-axis* injection, which, however, requires a change of the tips of the central plugs as well as minor changes of the central electrodes.

2. Derivation of the Root's solution

We take a coordinate system of Fig.1. In the absence of the magnetic field the central trajectory of the injected beam at $t = 0$ is easily seen to be (we take $v,\ R_E,\ E \geq 0$)

$$\boldsymbol{X}(t) \equiv \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = R_E \begin{pmatrix} 1-\cos(kt) \\ 0 \\ -\sin(kt) \end{pmatrix}$$
$$\text{with } \boldsymbol{E} = E \begin{pmatrix} \cos\theta \\ 0 \\ \sin\theta \end{pmatrix}, \text{ and } \theta \equiv kt , \tag{1}$$

where R_E is the radius of curvature under an electric field $E, k \equiv v/R_E$ and v is the velocity. Since $\boldsymbol{E}\cdot\boldsymbol{v} \equiv \boldsymbol{E}\cdot\dot{\boldsymbol{x}} = 0$ the magnitude of the volocity v is a constant of the motion.

When a uniform magnetic field $-\boldsymbol{e}_z B$ is applied, $z(t)$ and E_z are not affected, but $\boldsymbol{x}_H$ changes and $\boldsymbol{E}_H$ must be changed accordingly for conserving $\boldsymbol{E}\cdot\boldsymbol{v} = 0$; the suffix H denotes the horizontal component. For the sake of simplicity we use the following abbreviation.

$$c_n \equiv \cos(nkt) \equiv \cos(n\theta), s_n \equiv \sin(n\theta), \text{ and } ct_n \equiv \cot(n\theta). \tag{2}$$

We further define $\boldsymbol{E}_H^{\parallel}$ and $\boldsymbol{E}_H^{\perp}$, the former being parallel and the latter perpendicular to $\boldsymbol{v}_H$, respectively. For simplicity we *assume* $\boldsymbol{E}_H^{\perp} = 0$, then

$$\boldsymbol{E}_H \propto \boldsymbol{v}_H \equiv \dot{\boldsymbol{x}}_H \text{ ,hence } E_x = E_H \cdot \frac{\dot{x}}{v_H} \text{ and } E_y = E_H \cdot \frac{\dot{y}}{v_H} . \tag{3}$$

From $E_z = E\,s_1$, we have $E_H = E\,c_1$ and hence

$$E_x = E\,c_1 \frac{v_x}{v_H} = E\,c_1 \frac{\dot{x}}{v\,s_1} = \left(\frac{E}{v}\right) ct_1 \dot{x} \text{ and } E_y = \left(\frac{E}{v}\right) ct_1\, \dot{y} . \tag{4}$$

On the other hand the magnetic field $-B\boldsymbol{e}_z$ gives a horizontal force on the particle

$$F_x^B = -eB\dot{y} \text{ and } F_y^B = eB\dot{x} , \tag{5}$$

giving the equations of motion for x and y under the presence of $\boldsymbol{E}$ and $-B\boldsymbol{e}_z$;

$$m\ddot{x} = (eE/v)ct_1\,\dot{x} - eB\,\dot{y} ,\ m\ddot{y} = (eE/v)ct_1\,\dot{y} + eB\,\dot{x} . \tag{6}$$

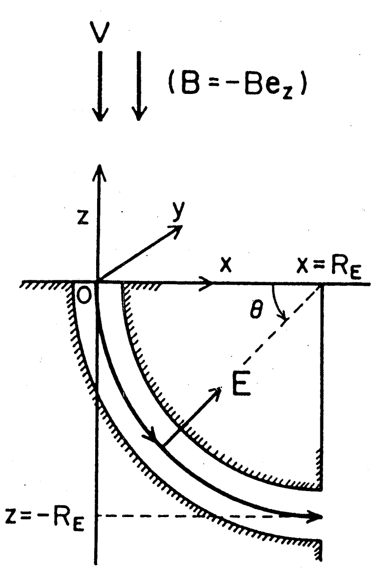

Fig.1. Deflection of central beam in the absence of B.

Defining the radius of curvature $R_{\rm M}$ under the presence of only B,

$$R_{\rm M} \equiv \frac{mv}{eB}\ , \text{ hence } eB/m = v/R_{\rm M} = \frac{v}{R_{\rm E}} \cdot \frac{R_{\rm E}}{R_{\rm M}} \equiv 2kK\ , \tag{7}$$

where $k \equiv v/R_{\rm E}$ and $K \equiv R_{\rm E}/(2R_{\rm M})$. From eqs.(6) and (7) we finally obtain the equations of motion for x and y ;

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = k \begin{pmatrix} {\rm ct}_1 & -2K \\ 2K & {\rm ct}_1 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} \equiv kM(t) \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} . \tag{8}$$

In eq.(8) the matrix $M(t)$ can be diagonalized by a unitary transformation;

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = U \begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} \text{ and also } \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = U \begin{pmatrix} \ddot{X} \\ \ddot{Y} \end{pmatrix}$$
$$\text{with } U \equiv \tfrac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} . \tag{9}$$

Eq.(8) then becomes

$$\begin{aligned} \begin{pmatrix} \ddot{X} \\ \ddot{Y} \end{pmatrix} &= kU^{+}M(t)U \begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} \\ &= k \begin{pmatrix} {\rm ct}_1 + 2iK & 0 \\ 0 & {\rm ct}_1 - 2iK \end{pmatrix} \begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} , \end{aligned} \tag{10}$$

i.e., *decoupled equations* for X and Y . Eq.(10) can be integrated as follows.

$$\ln|\dot{X}(t)| = \int_0^t k({\rm ct}_1 + 2iK)dt = \ln|{\rm s}_1| + 2ikKt\ , \tag{11}$$

giving

$$\dot{X} = C_{\rm X} e^{2ikKt}(e^{ikt} - e^{-ikt})/2i\ . \tag{12}$$

We take *from inspection* $\dot{Y} = 0$ and from eq.(9) obtain

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} &= \tfrac{C_{\rm X}}{\sqrt{2}} e^{2ikK} \cdot \tfrac{e^{ikt} - e^{-ikt}}{2i} \cdot \begin{pmatrix} 1 \\ -i \end{pmatrix} \\ &= \tfrac{C_{\rm X}}{2\sqrt{2}} \begin{bmatrix} -i({\rm c}_{2K+1} - {\rm c}_{2K-1}) + {\rm s}_{2K+1} - {\rm s}_{2K-1} \\ -i({\rm s}_{2K+1} - {\rm s}_{2K-1}) - {\rm c}_{2K+1} + {\rm c}_{2K-1} \end{bmatrix} . \end{aligned} \tag{13}$$

Taking the real part of eq.(13) and considering the initial and *final* conditions, $x(0) = \dot{x}(0) = y(0) = \dot{y}(0) = 0$ and $(\dot{x}^2 + \dot{y}^2)(kt = \theta = \pi/2) = v^2$, we arrive finally at the required solution;

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} &= \tfrac{v}{2} \begin{pmatrix} {\rm s}_{2K+1} - {\rm s}_{2K-1} \\ -{\rm c}_{2K+1} + {\rm c}_{2K-1} \end{pmatrix} \\ \text{and } \begin{pmatrix} x \\ y \end{pmatrix} &= \tfrac{R_{\rm E}}{2} \begin{pmatrix} -\frac{{\rm C}_{2K+1}}{2K+1} + \frac{{\rm C}_{2K-1}}{2K-1} - \frac{2}{4K^2-1} \\ -\frac{{\rm S}_{2K+1}}{2K+1} + \frac{{\rm S}_{2K-1}}{2K-1} \end{pmatrix} , \end{aligned} \tag{14}$$

which exactly coincides with the previous results [2,3], considering that we take $B_z = -B$. Before going further, we give complete forms of solution;

$$\begin{aligned} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \tfrac{R_{\rm E}}{2} \begin{pmatrix} -\frac{{\rm C}_{2K+1}}{2K+1} + \frac{{\rm C}_{2K-1}}{2K-1} - \frac{2}{4K^2-1} \\ -\frac{{\rm S}_{2K+1}}{2K+1} + \frac{{\rm S}_{2K-1}}{2K-1} \\ -2{\rm s}_1 \end{pmatrix} , \\ \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{x} \end{pmatrix} &= v \begin{pmatrix} {\rm s}_1{\rm c}_{2K} \\ {\rm s}_1{\rm s}_{2K} \\ -{\rm c}_1 \end{pmatrix} , \\ \text{and } \begin{pmatrix} E_{\rm x} \\ E_{\rm y} \\ E_{\rm z} \end{pmatrix} &= E \begin{pmatrix} {\rm c}_1{\rm c}_{2K} \\ {\rm c}_1{\rm s}_{2K} \\ {\rm s}_1 \end{pmatrix} . \end{aligned} \tag{15}$$

We note that $(E_{\rm x}, E_{\rm y}) \propto (\dot{x}, \dot{y})$ and that $\boldsymbol{E} \cdot \boldsymbol{v} = 0$.

3. Application to the CYRIC cyclotron

Before applying eq.(15) to our cyclotron, it is noted that the injected beam comes down along the mechanical axis of the cyclotron so that the entrance point is at $(x, y, z) = (0, 0, 0)$ and the exit point is at $(\rho, \phi, z) = (\rho_{\rm ex}, \Phi, -R_{\rm E})$ where the beam comes out horizontally in the median plane of the cyclotron, (ρ, ϕ, z) being the cylindrical coordinates; $\rho \equiv (x^2 + y^2)^{1/2}$ and $\Phi \equiv \phi_{\rm ex}$ is the total rotation angle of the beam orbit in the x-y plane.

From eq.(15) we note that we have two free parameters out of, *e.g.*, $R_{\rm M}, K$, and $R_{\rm E}$ because of the relation of $K = R_{\rm E}/(2R_{\rm M})$; we take $R_{\rm M}$ and K as independent parameters. As shown in Fig. 2, *as a crucial condition we require* that the beam leaving the inflector electrodes smoothly travel along a circle of radius $R_{\rm M}$ and passes perpendicularly through the slit in the radial part of the grounded housing of the inflector electrodes; the slit at a distance $R_{\rm s}$ faces the puller at $R_{\rm p}$ from the origin so that $R_{\rm s} = R_{\rm p} = 2.2$ cm for our cyclotron. For a smooth connection of the beam at the electrode exit we require

$$\gamma_{\rm in} = \gamma_{\rm out}\ , \tag{16}$$

where $\gamma_{\rm in}$ is the angle between $\boldsymbol{v}_{\rm H}$ and $\boldsymbol{e}_\phi$ at the exit point, *i.e.*,

$$\gamma_{\rm in} = \tan^{-1}\left(\frac{x\dot{x} + y\dot{y}}{x\dot{y} - y\dot{x}}\right)_{\theta=\pi/2} . \tag{17}$$

On the other hand, from the geometry of Fig. 2 we have at the exit point

$$\gamma_{\rm out} = \cos^{-1}\left(\frac{\rho_{\rm ex}^2 - R_{\rm s}^2 + 2R_{\rm M}R_{\rm s}}{2R_{\rm M}\,\rho_{\rm ex}}\right) . \tag{18}$$

Satisfying this condition (eqs.(16)-(18)) leaves one free parameter, *e.g.*, $R_{\rm M}$. We calculated numerically under the condition of eq.(16) various parameters as functions of $R_{\rm M}$ as shown in Fig. 3. Considering the central-region electrodes and the results given in Fig. 3, we chose $R_{\rm M}$ = 1.3 cm corresponding to $K = 1.1135$ as shown in Table

1; see also Fig. 2.

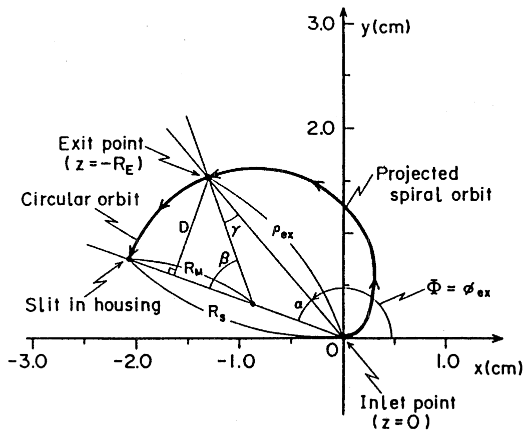

Fig. 2. Orbit projected to the x-y plane and definition of various parameters ($B \neq 0$); $R_s = 2.2$ cm and $K = 1.1135$.

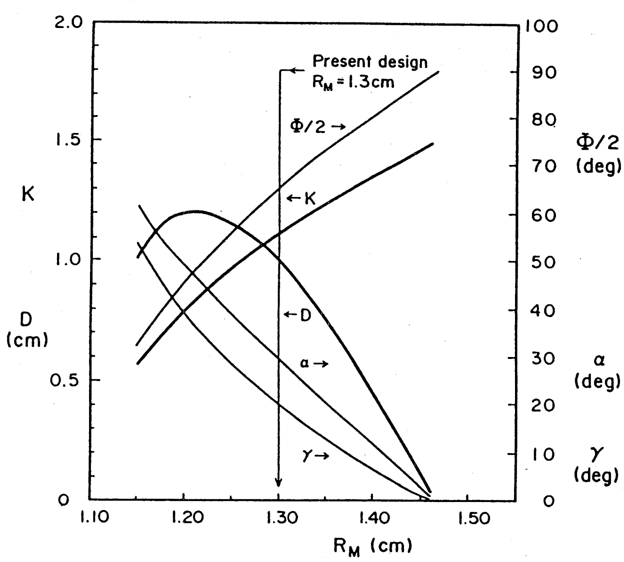

Fig. 3. Dependence of various parameters on R_M for R_s = 2.2 cm. Note that $\beta = \alpha + \gamma$ and $R_E = 2KR_M$, and the arrows indicate the scale to refer.

4. Conclusion

For the sake of understanding we gave a rather detailed derivation of the equation of the central trajectory of an axially injected ion beam (eq.(15)), and applied it to the CYRIC cyclotron, and proposed a set of design parameters compatible to the central-region structure of the cyclotron at the cost of least change of it; see Fig. 4. We would like to stress that the results of the survey calculation (Fig. 3) should be applicable to other cyclotrons with an appropriate scaling, *i.e.*, a normalization with respect to $R_s = R_p$.

The above discussion deals with only the central trajectory of the beam. Before manufacturing more ellaborate studies are needed resorting to a higher–order optical method or a numerical one.

Table 1. Design parameters of the spiral inflector

Parameters$^{a)}$	Design value
$R_s = R_p$	2.200 cm
R_M	1.300 cm
$K \equiv R_E/2R_M$	1.1135
$R_E{}^{b)}$	2.895 cm
ρ_{ex}	2.004 cm
$\Phi \equiv \phi_{ex}$	130.42°
$\gamma_{in} = \gamma_{out}$	19.99°
α	29.59°
β	49.58°
D	0.99 cm

a) See Fig. 3 and the text.
b) R_E essentially defines the height of the inflector electrodes.

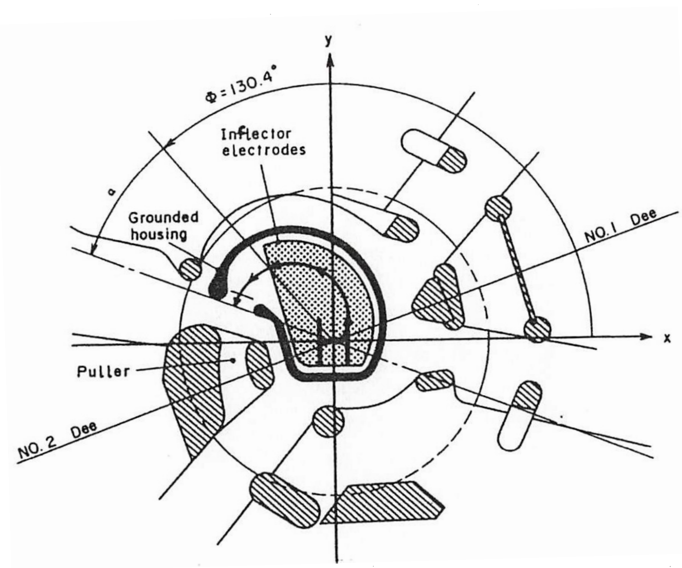

Fig. 4. Spiral inflector in the central region of the cyclotron.

References

[1] Honma T., Fujioka M., Goto A. and Shinozuka T., Proc. 9th Symp. Accel. Sci. Tech., Tsukuba, Japan (1993)300.

[2] Inoue M., RCNP-T-10, April (1983), in Japanese.

[3] Root L. M., Design of an inflector for the TRIUMF cyclotron, Master's thesis, Univ. British Columbia, April, 1972.

[4] Morita S., Fujioka M., Orihara H., et al., IEEE Trans. Nucl. Sci. NS-26 (1979)1930.