Transverse two-stream instability in a matched plasma channel

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Abstract

A relativistic electron beam magnetically self-focused in a plasma is subject to a transverse two stream or "hose" instability. Linear evolution is described in terms of a tune distribution characterizing the beam, and an effective transverse impedance determined by the beam and the plasma profiles. This model is compared to cloud-in-cell simulations of three-dimensional transport of a beam with a Bennett profile, through a matched plasma channel. In the limit of large skin-depth this instability appears to be the primary limitation on stable beam transport.

I. INTRODUCTION

In recent years the roles of plasma in accelerators have been the subject of many works [1-5] emphasizing that plasmas provide strong coupling, for focusing and for acceleration making use of relativistic electron beams. It is less widely appreciated that this strong coupling extends to strong *deflection* and *break-up* of the beam. In other arenas, plasma-coupled instabilities have motivated vigorous studies over many years, all concentrating on the resistive hose instability [6-8], or the filamentation instability [9,10]. In this literature there is scant reference to the transverse two-stream instability [11]. Here we will show that the electrostatic mode of beam break-up is typically more virulent than any other, in the limit of large skin-depth, the limit of primary interest for future accelerator applications.

II. ANALYSIS

We consider a relativistic electron beam with a Bennett-profile [12] charge density ρ_b, a function of the radial coordinate r and the beam coordinate $\tau \sim t-z/c$, where t is time, z is axial displacement and c is the speed of light. The beam propagates through a much denser plasma maintaining quasineutrality in equilibrium, provided the beam is several plasma periods in duration. Negligible plasma return current flows through the beam volume provided the plasma skin-depth is larger than the Bennett waist. Ion-motion and radiative effects are neglected. The equilibrium plasma is assumed stationary in τ, created rapidly by the beam head. We assume for simplicity that the plasma profile is matched to the beam.

To the Bennett equilibrium consider a *rigid* beam displacement $Y(z,\tau)$. Combining the Vlasov equation and the linearized cold fluid equations one can show that momentum conservation for the beam takes the form

$$\frac{\partial^2 \tilde{Y}}{\partial z^2} = k_s^2 \{ Z(p) - 1 \} \tilde{Y} \quad , \tag{1}$$

where a Laplace transform (indicated by the tilde) has been made in τ, with p the Laplace transform variable. The "slosh" wavenumber k_s is $k_s/k_\beta = 3^{-1/2}$ for the Bennett equilibrium, with k_β the wavenumber for small amplitude betatron oscillations. The normalized transverse impedance Z may be expressed as

$$k_s^2 Z(p) = -\frac{2\pi r_e c^2}{\gamma} \left\langle \frac{\partial}{\partial r}(r\psi) \right\rangle , \tag{2}$$

where r_e is the classical electron radius, γ is the Lorentz factor for the beam and the brackets denote an average over the unperturbed beam. The pinch potential ψ is determined from the solution of the elliptic equation in r,

$$\frac{1}{r}\frac{\partial}{\partial r} r\, \varepsilon \frac{\partial \tilde{\phi}}{\partial r} - \frac{\tilde{\phi}}{r^2} = \frac{\partial \rho_b}{\partial r} , \tag{3}$$

according to $\psi(r,p) = \hat{\psi}(r,\infty) - \hat{\psi}(r,p)$. The cold plasma dielectric function is just $\varepsilon = 1+\omega_e^2(r)/p(p+\nu)$ with ω_e the local angular plasma frequency and ν a phenomenological plasma electron collision rate.

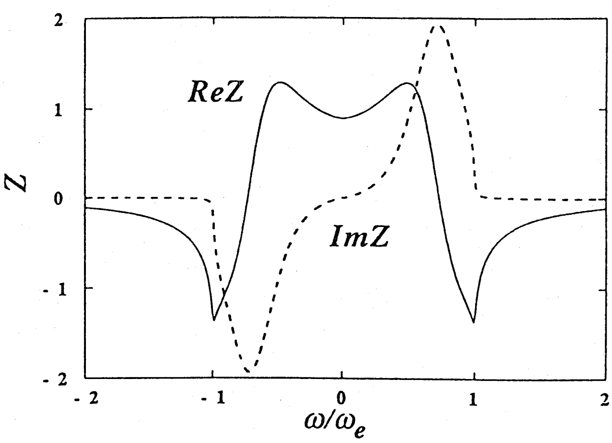

FIG. 1. The normalized transverse impedance of a Bennett plasma column, for a Bennett beam, is well-fit by a single-mode Lorentzian with Q~2.

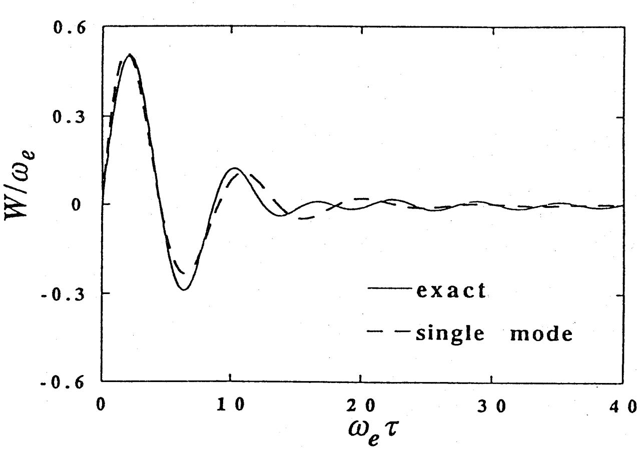

FIG. 2. The wakefield corresponding to the impedance of Fig. 1, overlayed with the single-mode fit.

In general it is not possible to solve for Z analytically; numerical solution is however straightforward and is depicted in Fig. 1 for the Bennett-profile plasma of interest here, with ν=0. The wake obtained with an inverse fast Fourier transform is shown in Fig. 2. This result is well-fit with a single-mode Lorentzian

$$Z(p) \approx \frac{\omega_L^2}{\omega_L^2 + p(p+\nu_L)}, \tag{4}$$

with resonant frequency $\omega_L \sim 0.71\omega_e$ and damping rate $\nu_L \sim 0.34\omega_e$. The corresponding wakefield (the inverse Laplace transform of Z) is overlayed in Fig. 2.

With this result it is straightforward to show that for $k_s z << \omega_e \tau$, the convecting peak in amplitude varies asymptotically as $Y \sim \exp(z/L_g)$ where $k_s L_g \sim 1$, i.e., growth proceeds rapidly, on the scale of the betatron period.

These results apply for short-range propagation, $k_\beta z \sim O(1)$. Over a long range, growth is diminished by phase-mixing in beam electron motion. This effect can be modelled following Lee's work on the resistive-hose problem[6]. The beam centroid is represented as an average, $Y = \int d\alpha\, g(\alpha) Y_\alpha$. The "mass distribution" $g = 6\alpha(1-\alpha)$ is normalized to unit integral, the dimensionless parameter α lies in the range [0,1], and the components satisfy

$$\frac{\partial^2 Y_\alpha}{\partial z^2} + \alpha k_\beta^2 Y_\alpha = \alpha k_\beta^2 Z(p) Y. \tag{5}$$

The corresponding dispersion relation is readily solved to establish that the saturation length L_s, scales according to $k_\beta L_s/\omega_L \tau \sim 3.4$, with amplitude at saturation $Y \sim \exp(\Gamma\tau)$, where $\Gamma \sim 0.9\omega_L$.

III. SIMULATION

To check this model we make comparison with a cloud-in-cell (CIC) simulation. This simulation advances the plasma variables (transverse position and nonrelativistic momentum) in the beam coordinate τ, with a leap-frog algorithm governed by the electrostatic potential.

The corresponding beam variables are advanced in z by a leap-frog algorithm, governed by the pinch potential. The neutralizing ion background is fixed. This formulation is consistent with (and limited to) large plasma skin-depth, an ultra-relativistic beam, and negligible radiative effects.

The saturation amplitude from the CIC simulation is shown in Fig. 3, overlayed with the model results, obtained by solving Eq. (5) in the time domain. A least-squares fit to the PIC results gives $\Gamma \sim \omega_L$. For a pulse 1.5 plasma periods in length, saturation occurs after growth by a factor of 1×10^2, for a uniform initial offset.

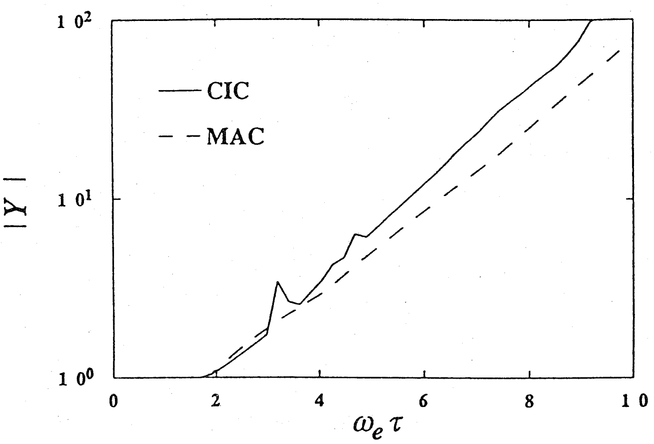

FIG. 3. Results for saturation amplitude from the numerical solution of the macroparticle model, the CIC simulation.

IV. DISCUSSION

Previous analyses of transverse stability have been performed in the limit of short plasma skin-depth where the effect of plasma return current is pervasive. Typically the plasma was assumed highly collisional, due to a low ionization fraction. In this limit, resistive hose growth has been a serious concern, with growth length scaling as $k_s L_g \sim \tau_D/\tau$, and τ_D the diffusion time-scale. For large skin-depth however, both resistive-hose and filamentation are minor compared to the strong electrostatic plasma resonance. Control of this beam beak-up mode favors damping of the wake, for example, with the plasma gradient considered here. Unfortunately this "cure" will also damp the often *desirable* longitudinal wake.

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