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Beam Tracking of a Compact Storage Ring (2)

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Abstract

A beam tracking with a numerical integration method has been studied for a design of a compact storage ring. Suitable numerical integration schemes for the beam tracking have been studied. Estimations of calculating speeds and accumulations of calculating errors have shown that a Runge-Kutta-Verner method and a Adams-Moulton's method are suitable for the beam tracking. Calculating results with various integration schemes have also shown that the numerical integration method can simulate an electron beam accurately when calculating errors are taken care of and tracking turns are restricted.

1 Introduction

Difficulties of simulating an electron's motion in a compact storage ring using a superconducting bending magnet are as follows:

1. non-linear terms of equations of motions must be treated, and
2. complicated three dimensional(3-D) magnetic fields (B_s, B_x, B_y) must be considered.

There are several methods to simulate the non-linear terms of equations of motion :

1. the Canonical integration method [1] ,
2. the Lie transformations method [2] , and
3. the numerical integration method [3] .

The numerical integration method has advantage of being able to treat exact equations of motion, being able to treat 3-D magnetic fields easily. The longitudinal magnetic fields (B_s) cannot be considerd with the other methods.

When non-symplectic algorithms are employed with a large truncation error, a phase space exhibits damping or exploring due to the noncanonical character of the integration algorithm. It is necessary to use a symplectic numerical integration scheme to simulate an electron's motion for a long time not to take care of a calculating error and a calculating step. Greenspan[4], Maeda[5], and J.M. SANZ-SERNA[6] have studied the schemes. J.M. SANZ-SERNA, for examples, had solved the condition of Runge-Kutta scheme which is satisfied with symplecticity. But these schemes take much CPU time and cannot be used for a beam tracking.

Considering practical uses, integration schemes which are not symplectic must be used to take care of calculating errors and to restrict tracking turns. In this paper suitable schemes of numerical integration for a beam tracking are discussed. This paper also discusses calculating accuracies of the numerical integration method.

2 Formulation

The study has been done with the tracking code 'PROVIDENCE'[7] which uses the numerical integration method. In this code, the equations of motion are accurately transformed to a simple form so as to fit to the numerical calculation as follows:

$$\begin{aligned} x'' &= \frac{C_1}{a}\left(-ay'B_s - x'y'B_x + (x'^2 + a^2)B_y\right) \\ &+ \frac{1}{\rho a}(2x'^2 + a^2) \end{aligned} \tag{1}$$

$$\begin{aligned} y'' &= \frac{C_1}{a}\left(ax'B_s + x'y'B_y - (y'^2 + a^2)B_x\right) \\ &+ \frac{1}{\rho a}(2x'y'). \end{aligned} \tag{2}$$

where ρ is the radius of curvature of the reference orbit (constant), $'$ denote the differentiation with s, and where a and C_1 are defined as follows:

$$a \equiv 1 + \frac{x}{\rho} \tag{3}$$

$$C_1 \equiv \frac{e}{mv}\sqrt{x'^2 + y'^2 + (1+\frac{x}{\rho})^2}. \tag{4}$$

These equations are precisely integrated by using adjusted step sizes. The 3-D magnetic fields (B_s, B_x, B_y) of a bending magnet are accurately simulated.

The numerical integrations and the estimations of truncation errors are done with the following methods:

1. a fourth-order Runge-Kutta method (R.K. method)[8] with a one-full step two half-step method,
2. a fifth-order Runge-Kutta-Fehlberg method (R.K.F. method)[8] with a embeded formula,
3. a fifth-order Runge-Kutta-Verner method (R.K.V. method)[9], with a embeded formula,
4. a fourth-order implicit Runge-Kutta method (I.R.K. method)[10] with a embeded formula,
5. a 12th-order Adams-Moulton's method (A.M. method)[11] with a predictor-corrector method,
6. a fifth-order Gear's BDF method (Gear's method)[11] with a predictor-corrector method.

R.K., R.K.F. and R.K.V. methods are explicit Runge-Kutta methods. These methods are efficient for non-stiff systems where the derivative evaluations are not expensive. I.R.K., A.M. and Gear's methods are implicit methods for stiff systems and a system of non-linear equations must be solved at each step.

3 Calculating results

3.1 Basic model

Calculating errors and calculating speeds are estimated with changing tolerances of the truncation errors. Electron's motions in a constant magnetic field are simulated. The simulating condition is as follows: an electron's energy, a bending radius, an initial position and tracking turns are $0.8GeV$, $0.593m$, $x_0 = 0.02m$ and 1000turns, respectively. An initial step size of the numerical integration is 1m, and the actual calculating step size is adjusted that the truncation error is within a tolerance. Table 1 shows beam positions and calculating times after 1000 turns tracking where the tolerance of the truncation error is $1.0 \times 10^{-11}m$. The calculatings were done with Sun Station10 (96Mips). An ideal position is $x = 0.02m$. Each calculating error and its sign are different even if the tolerance is the same value. The result that the absolute values of calculating errors are larger than the tolerance is due to accumulations of the truncation errors.

Figure 1 shows calculating speeds as a function of the absolute values of calculating errors after 1000 turns tracking. The figure leads that R.K.V. method and A.M. method are suitable for the numerical integration method and that I.R.K. method cannot use for a beam tracking because the simulating speed is very slow.

3.2 Mitsubishi's lattice

Estimations of calculating errors were also done with the compact storage ring whose magnetic fields were not constant. The calculating errors are considered to be larger than errors of the basic model, because beam position's errors of each step are a cause of a change of magnetic field. Lattice of the compact storage ring is shown in fig.2 which is in the Mitsubishi Electric Corporation. The energy and the bending radius are $800MeV$ and $0.593m$, respectively. The bending magnet of the ring consists of a pair of banana-shaped coils with a field index (about $-0.45m^{-2}$ at the center of the bending magnet) and an iron core.

Ideal values cannot be determined like the basic model. The calculating errors are estimated by the calculating results of R.K.V. and A.M. methods. These methods differ in accumulations of the calculating errors. The ideal values are considered to be within or very near the two calculated results. Tolerances of each method are so selected that calculating errors of the basic model are within 1×10^{-10}.

Table 2 shows the differences of the beam positons after 1000 tracking calculated with the two methods (R.K.V.− A.M.) in various initial conditions (x_0, y_0) and various multipole magnetic fields ($m = \frac{d^2B_y}{dx^2}, n = \frac{d^3B_y}{dx^3}$). Tune was selected $\nu_x = 1.33$, $\nu_y = 0.43$, respectively, because of simulating an unstable condition. When a beam position became larger than $x = 0.1m$ or $y = 0.1m$ before 1000 tracking, the tracking was ended (case 9). Table 2 leads that the simulation of the numerical integration method has a good accuracy of the order of 1×10^{-9}, when calculating errors of the basic model are within 1×10^{-10} and tracking turns are 1000.

4 References

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Table 1
Beam positions and calculating times after 1000 turns tracking where the tolerance of the truncation error is $1.0 \times 10^{-11} m$.

Method	Position[m]	Time [s]
R.K. method	$0.02 - 1.08 \times 10^{-11}$	130.1
R.K.F. method	$0.02 + 6.97 \times 10^{-11}$	41.7
R.K.V. method	$0.02 - 7.04 \times 10^{-11}$	21.2
I.R.K. method	$0.02 + 2.83 \times 10^{-5}$	87600
A.M. method	$0.02 + 7.53 \times 10^{-8}$	17.2
Gear's method	$0.02 - 1.29 \times 10^{-6}$	28.0

Table 2
Differences of the beam positons (Diff.x,Diff.y) after 1000 turns tracking calculated with the two methods (R.K.V.– A.M.). Initial points are $\beta_x$maximum and $\beta_y$minimum.

case	$x_0[mm]$	$y_0[mm]$	$m[\frac{T}{m^2}]$	$n[\frac{T}{m^3}]$	Diff.x[m]	Diff.y[m]	turns
1	25.0	5.0	0.0	0.0	-4.97×10^{-11}	-1.26×10^{-11}	1000
2	25.0	5.0	5.0	0.0	-1.47×10^{-11}	-2.14×10^{-11}	1000
3	25.0	5.0	5.0	100.0	-2.88×10^{-11}	-1.51×10^{-10}	1000
4	25.0	5.0	10.0	0.0	1.85×10^{-11}	-3.26×10^{-11}	1000
5	25.0	5.0	10.0	100.0	-1.28×10^{-11}	-8.61×10^{-11}	1000
6	50.0	10.0	0.0	0.0	-1.21×10^{-10}	-1.62×10^{-11}	1000
7	50.0	10.0	5.0	0.0	-7.04×10^{-11}	-6.32×10^{-11}	1000
8	50.0	10.0	5.0	100.0	-8.34×10^{-11}	1.33×10^{-10}	1000
9	50.0	10.0	10.0	0.0	1.99×10^{-12}	1.97×10^{-12}	50
10	50.0	10.0	10.0	100.0	1.11×10^{-11}	-7.04×10^{-10}	1000

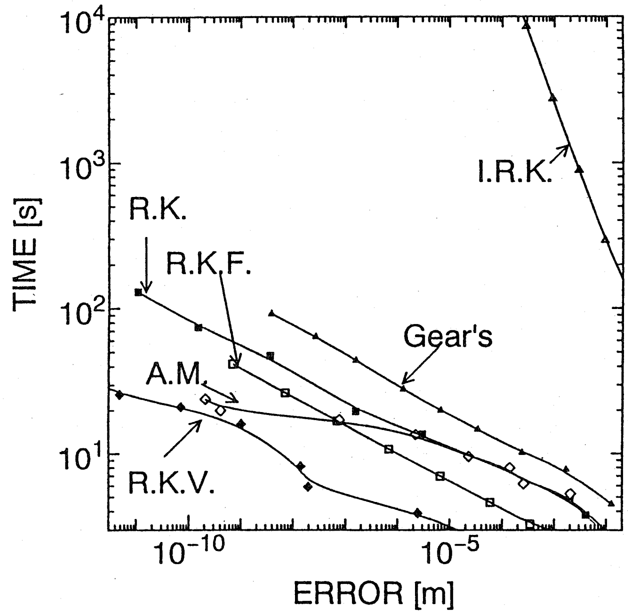

Figure 1 Calculating speeds as a function of the absolute values of calculating errors after 1000 turns tracking.

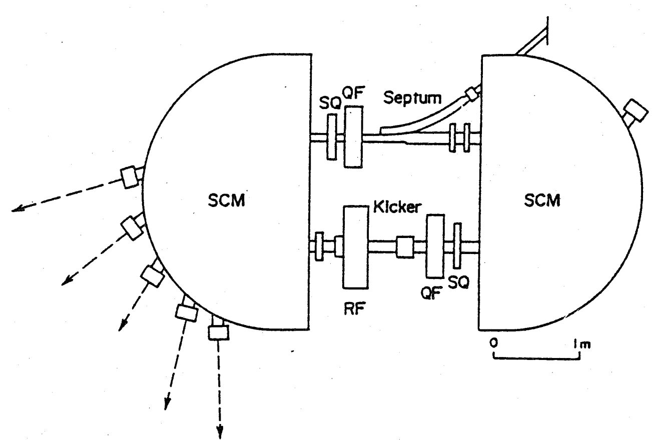

Figure 2 Schematic drawing of the Mitsubishi's SR ring.