BUNCH LENGTHENING IN TRISTAN AR

K. Nakajima, K. Ebihara, A. Ogata and K. Satoh
National Laboratory for High Energy Physics
Oho-machi, Tsukuba-gun, Ibaraki-ken, 305, Japan

Summary

A phenomenon of bunch lengthening in TRISTAN AR has been investigated by measuring the bunch length of a single electron bunch. It is found that the current dependence of bunch length is in agreement with a potential well distortion model below a threshold and with a mode coupling theory above it. The observation of longitudinal bunch oscillation modes confirms the fact that anomalous bunch lengthening is caused by mode coupling instabilities driven by a strong coupling impedance due to a number of RF cavities and bellows in the ring.

Introduction

A bunch lengthening phenomenon has been a puzzle in high energy particle accelerators or storage rings. For a electron storage ring, the particles have a Gaussian distribution in longitudinal line density due to equilibrium between quantum excitation and radiation damping[1]. In most electron storage rings one observes an increase of bunch length as increasing beam currents. Two types of bunch lengthening mechanism have been presented.

The potential well distortion due to the interaction of space charge with inductive surroundings leads to bunch lengthening and to a shift of the incoherent synchrotron frequency[2,3]. Above a certain threshold current anomalous bunch lengthening occurs with increasing in the rate of bunch lengthening and in energy spread[4-6]. This phenomenon has been explained by the onset of longitudinal instability in a single bunched beam at very high frequencies, which is called "microwave instability". A number of theoretical models[7-10] to explain the bunch lengthening phenomenon have been proposed and obtaining a qualitative agreement with the observations. However, there seems to be no satisfactory theory which can predict threshold currents and bunch lengths quantitatively for a given coupling impedance of the ring.

In this work, the bunch lengths for various beam energies and synchrotron frequencies have been measured very accurately by using the streak camera. The bunch shapes are calculated from the potential well distortion model by assuming the longitudinal coupling impedance of the ring, and compared with the experimental data below thresholds. Above thresholds the data have been compared with predictions of the bunch lengthening threshold derived from a mode coupling theory used to explain the microwave instability. The existence of the mode coupling has been observed by the spectroscopy of longitudinal bunch oscillation modes. From the scaling property on the instability, threshold currents of the bunch lengthening have been obtained, which are useful to predict the instability thresholds and bunch lengths for any machine parameters.

Bunch Length Measurements

The longitudinal line density of electron bunch can be viewed by focusing a visible synchrotron light on the streak camera system. Details of this system are reported elswhere[11]. The intensity of a streak shot projected on the longitudinal axis is digitized with time resolution of 2 ps and read out by the computer connected to the streak camera. The rms length of a single bunch is determined by fitting the digitized data of bunch shape to a Gaussian distribution with backgrounds. A typical bunch shape is shown in Fig.1. It is noted that the measured bunch shape tends to deviate from a purely Gaussian distribution at high currents and the modulated structure is seen at times. It is not clear whether the modulation on the bunch shape is caused by bunch oscillation due to the beam instability or by statistical fluctuation in scanning of the streak camera. The measurement error is dominated by uncertainties in time calibration of the streak camera and fluctuations of bunch shape in shot by shot. The overall systematic error is estimated to be ~5% on the average.

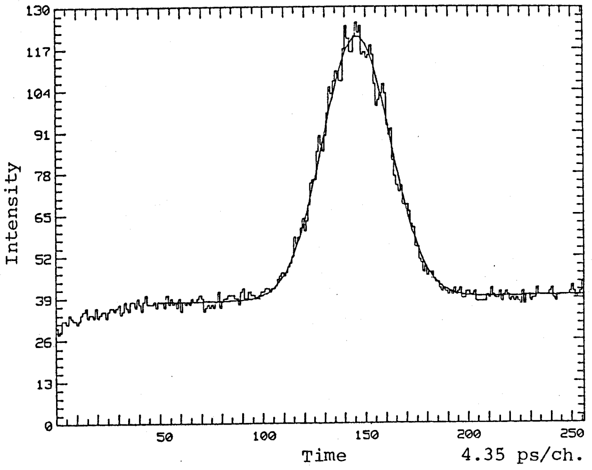


Fig. 1 A typical bunch shape taken with the streak camera. The solid curve is a fit to data with a Gaussian peak plus a linear background.

The data of bunch shape have been taken as a function of wide-ranging values of average current I, total RF cavity voltage, and beam energy E. In order to calibrate the total RF cavity voltage, the frequency f_s of synchrotron oscillation has been measured near zero currents (<1mA) by modulating RF amplitudes. The synchrotron tune is determined by $\nu_s = f_s/f_0$, where f_0 is the revolution frequency 794.65 kHz. The bunch lengthening factor is obtained by dividing σ by the natural bunch length σ_0, calculated from machine parameters in the classical way[1].

Some of the data on the bunch lengthening are plotted in Fig. 2 as a function of average beam current at (a) 2.55 Gev and (b) 3.00 GeV, respectively. In these figures, it is visibly noted that the rate of bunch lengthening changes at a certain threshold current. The bunch length increases slowly below the threshold and increases abruptly above it. In Fig.2(a) one can see an another abrupt blow-up arround 30mA. These data suggest that below the threshold current the potential well distortion may produce a slowly varying bunch lengthening and above it the dominant mechanism of the bunch lengthening results from longitudinal bunched beam instabilities.

Potential Well Distortion

A potential well distortion model[12] explains qualitatively the bunch lengthening mechanism in the following; The beam-induced fields generated by the interaction of a bunched beam with surroundings (cavities and vacuum chambers) lead to a distortion of RF voltage seen by the particles and cause a shift in the incoherent synchrotron frequency. This lengthens (or shortens) the bunch length unless the energy spread is affected.

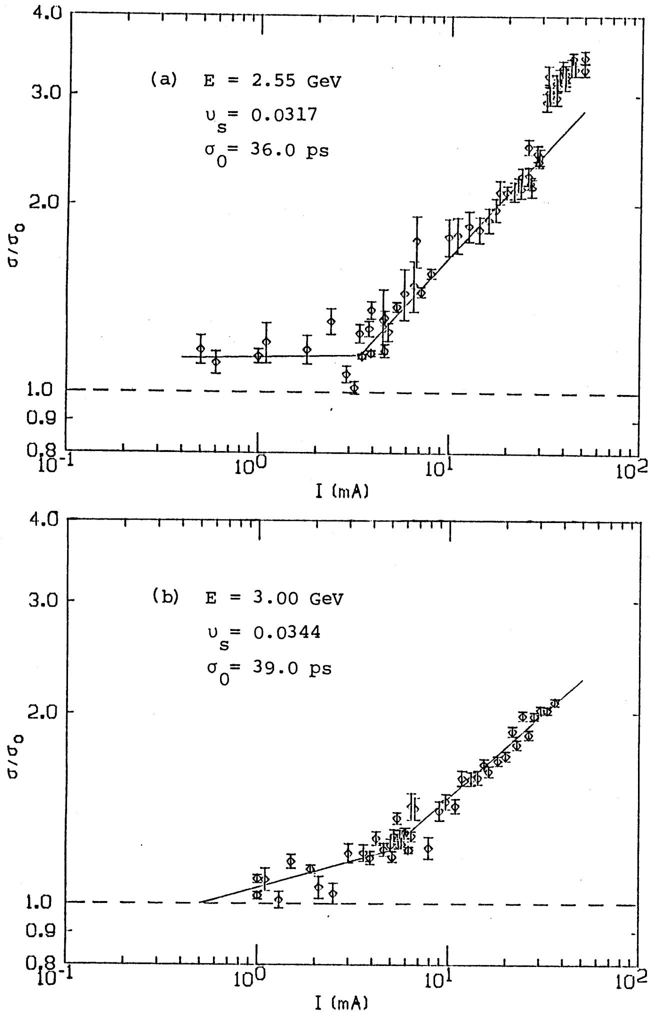


Fig. 2 Bunch lengthening as a function of beam current.

The effect on the bunch shape of potential well distortion has to be investigated by the expression[13] for the current distribution I(t), given by

$$I_n(t) = K\exp[-\frac{t^2}{2\sigma^2} - \frac{G}{\sigma^3}\int_0^\infty s(\tau)I_n(t-\tau)d\tau] \qquad (1),$$

where $I_n(t)=I(t)/I_p$ and $I_p=I/(f_0\sqrt{2\pi}\sigma)$ is the peak current for the unperturved Gaussian bunch. Here $G=\alpha I/(f_0{}^2\upsilon_s{}^2E)$ is a scaling parameter of the strength of potential well distortion. The constant K is chosen to conserve total charge so as to satisfy

$$\int_{-\infty}^{\infty} I_n(t)dt=\sqrt{2\pi}\sigma \qquad (2).$$

The wake function $s(\tau)$ gives the response at time τ to a unit current step at $\tau=0$. The self-consistent solution of equation (1) can be numerically computed by assuming the unperturbed Gaussian distribution. The computed rms bunch lengths are compared with the data measured at E=4.00 GeV and υ_s=0.0289 in Fig.3 as a function of the current below a threshold 20mA. Note that the computed values are in good agreement with the measurement.

Scaling Law for Bunch Lengthening

The scaling law[14] for the bunch length predicts that the bunch length σ_z is a certain function of a single scaling parameter $\xi=\alpha I/\upsilon_s{}^2E$ above the bunch lengthening threshold. All data taken for various υ_s, E and I but a fixed momentum compaction α are plotted in Fig.4 as a function of the scaling parameter ξ. The scaling law seems to be good within the measurement error. However, notice that the σ_z cannot be expressed by a single function of the scaling parameter because an abrupt blow-up occurs around ξ =150(mA/GeV). Fig.4 indicates that the bunch lengthening data above thresholds are parametrized by

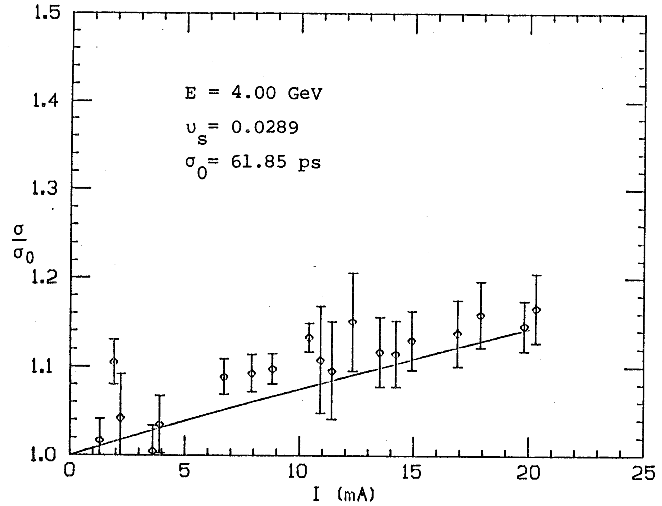


Fig. 3 Bunch lengthening below the threshold current. The solid curve shows the calculation from Eq. (1).

$$\sigma_z(\text{cm})=0.55\xi^{0.31}(\text{mA/GeV}) \text{ for } \xi<150 \text{ (mA/GeV)} \qquad (3a),$$

$$\sigma_z(\text{cm})=0.70\xi^{0.31}(\text{mA/GeV}) \text{ for } \xi>120 \text{ (mA/GeV)} \qquad (3b).$$

Since the threshold falls on the same scaling law, the threshold value of ξ is given by $\xi_{th}=7.0\sigma_{th}{}^{3.25}$ for σ_{th}<3cm, where σ_{th} is the bunch length at the threshold. The data on the bunch length at the thresold give $\sigma_{th}/\sigma_0=1.14$, independent of ξ. It indicates that above the threshold the unperturbed bunch length is slightly legthened by the effect of potential well distortion. By using these results, the threshold current can be predicted by

$$I_{th}(\text{mA})=6.82\text{x}10^{-4}\upsilon_s{}^{-1.25}E^{4.25}(\text{GeV}) \qquad (4),$$

for any set of E and υ_s with a fixed α=0.01252.

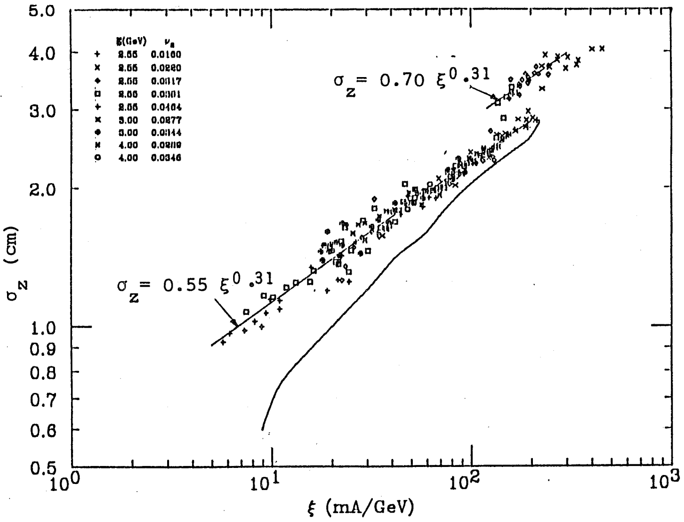


Fig. 4 Bunch lengths above the threshold currents as a function of scaling parameter $\xi = \alpha I/\upsilon_s{}^2E$. The solid curve shows the prediction of the bunch lengthening threshold calculated from the mode coupling theory.

Mode Coupling Instability

The scaling property suggests that the bunch lengthening could be explained by the mode coupling instabilities in the following: At zero current the frequency of the longitudinal coherent oscillation mode is simply multiples of the incoherent synchrotron frequency. As the current increases, the coherent mode frequencies shift due to the impedance of the beam surroundings and then two mode frequencies merge at the threshold current. As the current exceeds the threshold, the mode frequencies turn to be imaginary and the beam becomes unstable. Above the threshold, the bunch will be legthened untill equilibrium with radiation damping can be reached and the beam becomes stable again so that the current stays at a new stability threshold corresponding to the bunch length.

The mode coupling theory developed in ref.15 has been applied to the calculation of the bunch lengthening threshold in TRISTAN AR. The solid curve in Fig.4 shows the result for 6 azimuthal modes and 18 radial modes. The calculation has been done with the longitudinal coupling impedance calculated by a program BCI for RF cavities and bellows in the ring. Note that in the calculated curve of Fig.4 the thresholds are determined by four mode couplings depending on the bunch length. In Fig.5 the cohrent frequencies of the radial modes are plotted as a function of the current. It is found that one of the radial modes with the dipole mode frequency and one of them with quadrupole mode frequency shift as the current increases and they merge at 3.6 mA. The calculation gives slightly smaller bunch legthening compared to the measurments. If we take account of the bunch lengthening of the unperturbed distribution due to potential well distortion, the predictions of the mode coupling theory will fit to the experimental data fairly well. It is expected that the theory may predict the fast blow-up at the higher scaling parameters $\xi \simeq 150$ if the calculation is improved.

An experimental evidence for the mode coupling instability has not been found in the bunch lengthening study at the existing storage rings. In order to find the existence of radial modes and the occurrence of their mode coupling, the output from a position monitor electrode is directly coupled to a wide band spectrum analyser (ANRITSU MS710A) with the frequency range of 10 MHz to 23 GHz. The spectrum is observed at 4.46 GHz as varying the current at E=2.55 GeV and $\nu_s=0.030$, as shown in Fig. 6. Since the frequency spectrum for the dipole, quadrupole and sextupole mode peaks at 4.19, 5.92 and 7.25 GHz, respectively, for $\sigma_0=38$ ps, we can find a signal of the coherent oscillation due to the mode coupling between the dipole and quadrupole modes around 5 GHz. In Fig.6(a) we find the merging of radial modes between the dipole and quadrupole modes at 8mA. At 12mA the merging between the quadrupole and sextupole modes can be seen in Fig.6(b). Above an abrupt increase of the bunch length at 30mA, many multipole modes appear with a widened spread of mode frequency in Fig.6(d). It seems to be a re-combining of radial modes splitted from the unperturbed modes. A widening of the frequency spread may cause the abrupt blow-up of bunch at very high currents.

Conclusion

Through the measurements of the bunch length and the observation of bunch oscillation in TRISTAN AR, it is concluded that anomalous bunch lengthening is essentially caused by the instability due to the mode coupling between two radial modes of longitudinal bunch oscillation. The mode coupling instability is driven by the broadband impedance due to a large number of resonances in the RF cavities and the bellows. The bunch lengthening below a threshold current results from the potential well distortion induced by the same longitudinal coupling impedance of the ring. The experimental data on the threshold current can be simply parametrized by the machine parameters.

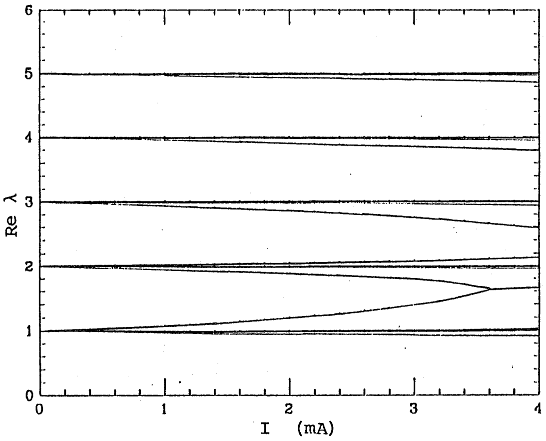
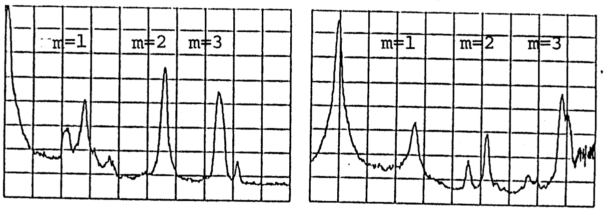
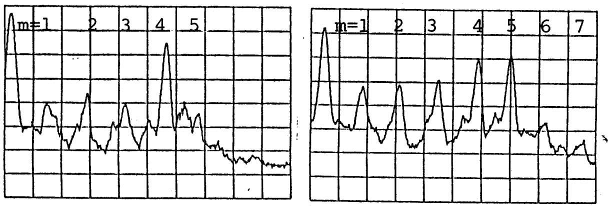


Fig. 5 The real part of the coherent frequency λ in unit of synchrotron frequency as a function of beam current. The bunch length is 1 cm.



(a) 8 mA, 10 kHz/div. (b) 12 mA, 10 kHz/div.



(c) 25 mA, 20 kHz/div. (d) 30 mA, 20 kHz/div.

Fig. 6 coherent mode frequency spectrum around 4.46 GHz, taken at E = 2.55 GeV, ν_s = 0.030 and various currents.

Acknowledgments

We wish to thank all the other members of the TRISTAN AR operation group, in particular, S. Kamada.

References

1. M. Sands, SLAC-121(1970).
2. C. Pellegrini, A.M. Sessler, Nuovo Cimento 3A(1971) 116.
3. F. Sacherer, PEP Summer Study Note No.45(1973).
4. P.B. Wilson et al., IEEE Trans. Nucl. Sci. NS-24(1977) 121.
5. R.D. Kohaupt, Proceedings of the Xth Intern. Conf. on High Energy Accelerators, Vol.2(1977) 43.
6. J.C. Denard et al., IEEE Trans. Nucl. Sci. NS-28(1981) 2474.
7. P. Channel, A.M. Sessler, N.I.M. 136(1976) 473.
8. M. Month, E. Messerschmid, N.I.M. 141(1977) 1, 144(1977) 119.
9. F.J. Sacherer, IEEE Trans. Nucl. Sci. NS-24(1977) 1393.
10. B. Zotter, IEEE Trans. Nucl. Sci. NS-28(1981) 2602.
11. A. Ogata, Y. Mizumachi, these proceedings.
12. B. Zotter, CERN SPS/81-14(1981).
13. K.L.F. Bane, P.B. Wilson, IEEE Trans. Nucl. Sci. NS-24(1977) 1485.
14. A.W. Chao, J. Gareyte, SPEAR-197/PEP-224(1976).
15. T. Suzuki, K. Satoh, Y. Chin, Particle Accelerators, Vol.13(1983) 179.