DESIGN STUDY OF THE THIRD ORDER RESONANCE EXTRACTION SYSTEM AT TARN II

M. Takanaka and A. Itano
National Institute of Radiological Sciences
Anagawa 4-9-1, Chiba-city, Chiba 260, Japan

A. Noda, T. Tanabe and H. Tsujikawa
Institute for Nuclear Study, University of Tokyo
Midori-cho 3-2-1, Tanashi-city, Tokyo 188, Japan

Abstract

Stress has been put on for a circulating beam to have as large emittance at the time of extraction as possible in the design study. The maximun separatrix area at the time is 54π mm*mrad for 1.3 GeV proton beam with ΔP/P= ±0.2 %. The emittance of the extracted beam is about 0.3π mm*mrad.

Introduction

The tune value Q of TARN II is 1.75 for acceleration operation, and is considered 1.75 or 2.25 for electron cooling operation. Slow beam extraction is carried out by using the third order resonance. In the paper, we present the design study of the slow beam extraction system for acceleration operation.

Third order resonance extraction has been successfully carried out at LEAR[1]. Chromaticity adjustment has been important for the good beam extraction. At TARN II the transverse emittance is not shrunk by acceleration so much as at high energy accelerators. Stress is put on for a circulating beam to have as large emittance at the time of extraction as possible in design study at TARN II.

Beam Extraction System

The beam extraction system consists of four elements as shown in Fig. 1:

i) four bump magnets which distort the closed orbit

ii) sextupole magnets as a chromaticity adjuster and as a resonance exciter

iii) main dipole and quadrupole magnets as a closed orbit shifter

iv) an electrostatic septum(ES) and three magnetic septa(MS) as a extracter.

As a circulating beam at the time of multiturn injection must be given a large open horizontal space, the nearest thing to the beam, the anode plate of ES is set at x=84 mm as far from the central orbit as possible. The closed orbit is distorted between the bending section just before ES and the next bending section by four bump magnets for the reason that an extracted beam should go into ES parallel to the anode plate on the anode side, and in consideration of turn separation and of the horizontal space for a circulating beam. The bump magnets are four with backleg winding coils of 24 main dipole magnets.

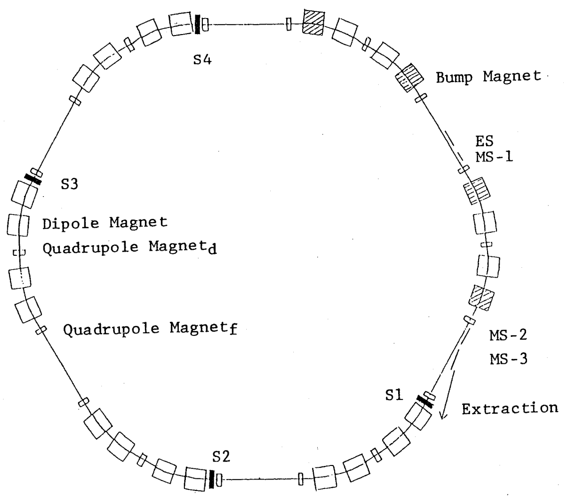

Fig. 1 Extraction System
Sextupole magnets only as a chromaticity adjuster are not shown here.

Description of sextupole magnets is given in a later Section.

On beam extraction process, at the first, after acceleration Q-value is changed from the injection value 1.75 to a value near the third order resonance. Secondly, when the momentum corresponding to the central orbit, or the on-central-orbit momentum P0 is gradually decreased by decrement of main dipole magnet's field with tracking of quadrupole magnet's field, Q-value of the beam is shifted toward/away from the resonance value for chromaticity negative/positive. The separatrix is roughly speaking shrunk/enlarged. In case of the shrinkage the beam can be extracted.

The beam with turn separation 10 mm at most is extracted with an ES, a MS at a straight section and two MS's at the next straight section. The acceptance at ES is 10mm * 2mrad. Parameters of the septa are listed in Table 1[2].

Table 1
Parameters of the Septa

type	septum thickness (mm)	field (kV/cm),(kG)	length (m)	kick (mrad)
ES	0.05	40	1.0	1.5
MS-1	1	1	0.55	8.0
MS-2	9	5	1.0	76.4
MS-3	35	16	1.3	302.9

Dependence of the Separatrix Size on the Closed Orbit Displacement

A particle with Q-value near the resonance($Q=Q_{res}+\Delta Q$) is considered to recieve a kick from a sextupole magnet, whose strength is $\Delta X'=SX^2$, every revolution. The closed orbit of the particle at the sextupole magnet is positioned $X_d=\eta_s*\Delta P/P0$ from the central orbit, η_s being the dispersion at the sextupole magnet, and ΔP momentum displacement from the on-central-orbit momentum P0. After three revolutions the horizontal phase space point (X,X') at the azimuthal of the magnet is shifted by

$$\Delta X=[(\varepsilon-3SX_d)+\frac{3}{2}S(X-X_d)](X'-X_d')$$
$$\Delta X'=-(\varepsilon-3SX_d)(X-X_d)+\frac{3}{4}S[(X-X_d)^2-(X'-X_d')^2] \quad (1)$$

where $\varepsilon=6\pi\Delta Q$,
ignoring terms with $(\Delta Q)^2$ and higher as was described by Barton[3].
Taking the time for three revolutions as a unit time, and using

$$\Delta X=\frac{dX}{dt}=\frac{dH}{dt}$$
$$\Delta X'=\frac{dX'}{dt}=-\frac{dH}{dt} \quad (2)$$

we derive the equations of motion from the Hamiltonian

$$H=\frac{\varepsilon-3SX_d}{2}[(X-X_d)^2+(X'-X_d')^2]+\frac{S}{4}[3(X-X_d)(X'-X_d')^2-(X-X_d)^3]. \quad (3)$$

When H has the value

$$\frac{8}{27S^2}(\varepsilon-3SX_d)^3 \tag{4}$$

it factors into three straight lines

$$[\frac{S}{4}(X-X_d)+\frac{1}{6}(\varepsilon-3SX_d)][\sqrt{3}(X'-X_d')+(X-X_d)-\frac{4(\varepsilon-3SX_d)}{3S}] * [\sqrt{3}(X'-X_d')-(X-X_d)+\frac{4(\varepsilon-3SX_d)}{3S}]=0 \tag{5}$$

giving the separatrix. The distance between the stable fixed point and the unstable fixed point is

$$r=\frac{4\varepsilon}{3S}-4X_d=\frac{8\pi\Delta Q}{S}-4X_d \; . \tag{6}$$

When a particle with P0 has Q-value Q0, a particle with P0+ΔP has Q-value

$$Q=Q0+\xi\, Q0\frac{\Delta P}{P0}\;, \tag{7}$$

ξ being the horizontal chromaticity.
Thus,

$$\Delta Q=Q0-Q_{res}+\xi\, Q0\frac{\Delta P}{P0}\;. \tag{8}$$

The size of the separatrix or r has dependence on
i) operation tune value Q0
ii) chromaticity
iii) closed orbit displacement from the central axis of the sextupole magnet.

For Good Extraction

At the entrance of ES, a beam near the anode must have no slope against the anode plate. When the slope is declined outwards, the most outside circulating beam hits the anode and is lost. When inward, so does the inside extracted beam. We make discussions, following the previous Section. To make beam loss as small as possible, it is important to make coincide outgoing trajectories of beams with different emittances at ES and to inject the beam parallel to the anode plate on the anode side[4].

In Fig. 2, separatrices and outgoing trajectories at ES are shown on the normalized phase space on condition that the trajectories are overlapped each other. Without the condition, stable fixed points are always on the line $X'=\alpha X$. This condition is made by
i) defining the intersecting point of the trajectory and line $X'=\alpha X$
ii) having a zero emittance separatrix at the intersecting point.

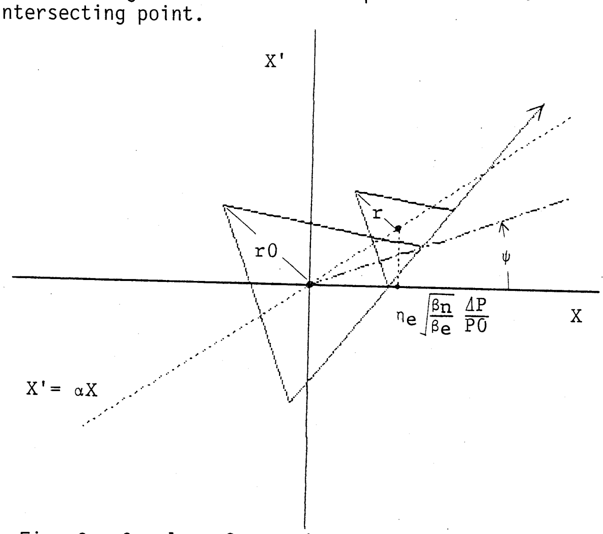

Fig. 2 Overlap of outgoing trajectories on the normalized phase space

The trajectory belonging to the beam with ΔP=0 is given by

$$X'=\frac{\cos\psi+\sqrt{3}\sin\psi}{\sqrt{3}\cos\psi-\sin\psi}X-\frac{r0}{\sqrt{3}\cos\psi-\sin\psi}\;, \tag{9}$$

where r0 is r for ΔP=0, and ψ a phase advance from ES to the resonance exciter.
The X coordinate of the intersecting point is given by

$$X=\frac{r0}{(1-\sqrt{3}\alpha)\cos\psi+(\sqrt{3}+\alpha)\sin\psi}\;. \tag{10}$$

When the X or a zero emittance point is defined, ψ is seeked.

The momentum displacement ΔP of the zero emittance beam is written as

$$\eta_e SQRT(\frac{\beta_n}{\beta_e})\frac{\Delta P}{P0}=\text{right hand side of Eq.(10)} \tag{11}$$

where η_e and β_e are dispersion and beta functions at ES, respectively, and β_n normalized beta function. Thus, the zero emittance beam is realized by making r zero:

$$\xi=\frac{Q_{res}-Q0}{Q0}\,\frac{P0}{\Delta P}+\frac{S\eta_s}{2\pi Q0}\,SQRT(\frac{\beta_n}{\beta_s})\;. \tag{12}$$

The point where the trajectory crosses the septum of ES has some X' value. The closed orbit is distorted by using bump magnets in order that the point should have the same X' value in practice as the anode plate has.

Summation Expression of the Sextupole Magnets

Sextupole magnets are installed on the ring in order to adjust chromaticity and excite the third order resonance.

Using the complex notation Zb=X-Xd+i(X'-Xd'), Eq. (1) is rewritten as

$$\Delta Zb=3i[(-2\pi\Delta Q+SX_d)Zb+\frac{S}{4}Zb^{*2}]\;. \tag{13}$$

When the phase space point(X,X') at phase advance ψ_j is expressed by using $Zb^*e^{-i\psi_j}=X-Xd+i(X'-Xd')$, it is shifted by the sextupole magnet S_j at the phase advance,

$$\Delta(Zbe^{-i\psi_j})=3i[(-2\pi\Delta Q+S_jX_{dj})Zbe^{-i\psi_j}+\frac{S_j}{4}Zb^{*2}e^{2i\psi_j}]. \tag{14}$$

The shift becomes at the phase advance 0

$$3i[(-2\pi\Delta Q+S_jX_{dj})Zb+\frac{S_j}{4}e^{3i\psi_j}Zb^{*2}]. \tag{15}$$

Therefore, after three revolutions the phase space point at the phase is shifted under sextupole magnets by

$$3i[(-2\pi\Delta Q+\Sigma S_jX_{dj})Zb+\Sigma\frac{S_j}{4}e^{3i\psi_j}Zb^{*2}]. \tag{16}$$

An effective sextupole magnet with S can be considered to be located at phase advance ψ instead of the sextupole magnets:

$$Se^{3i\psi}=\Sigma S_je^{3i\psi_j}\;. \tag{17}$$

Equation (16) holds at all phases $=\psi$ (mod $2\pi/3$).
If we need no dependence of the separatrix size on the closed orbit displacement,

$$\Sigma S_jX_{dj}=0 \tag{18}$$

must be kept.

Four(S1, S2, S3 and S4) from installed sextupole magnets are selected as a resonance exciter. β_s's (x and y) and η_s's are equal, respectively, β_{sx}'s large, phase advances at them symmetrical for ES position, and

$\cos(3\psi_j)$ has about 1 at S1, S4, and a value aroud 0 at S2 and S3, respectively, as the phase advance is let 0 at ES. Strength S_j which they additionally have after acceleration are let meet following demands:

i) no change in chromaticity (x and y)

ii) no dependence of the separatrix size on the closed orbit displacement.

As beta (x and y) and dispersion functions at the magnets are equal, respectively, the demands are

$$\Sigma S_j=0 \ . \qquad (19)$$

Beam Trace on Computer Simulation

A beam is assumed to have 400π mm*mrad at most after multiturn injection. After acceleration, the momentum spread is $\Delta P/P=\pm 0.2$ %. Emittance at the time is dependent on ion species, charge mass ratio and energy. We have seeked the case where the circulating beam can have considerably large emittance for maximum energy 1.3 GeV proton beam on computer simulation. Following requirement has been imposed upon the beam:

i) turn separation 7 mm for the beam with $\Delta P/P0=0.2\%$

ii) intersecting point (50 mm, 0 mrad)

iii) the phase advance 0 of effective sextupole manget.

The four sextupole magnets have been treated as thin lenses. A beam has been traced on the normalized phase space.

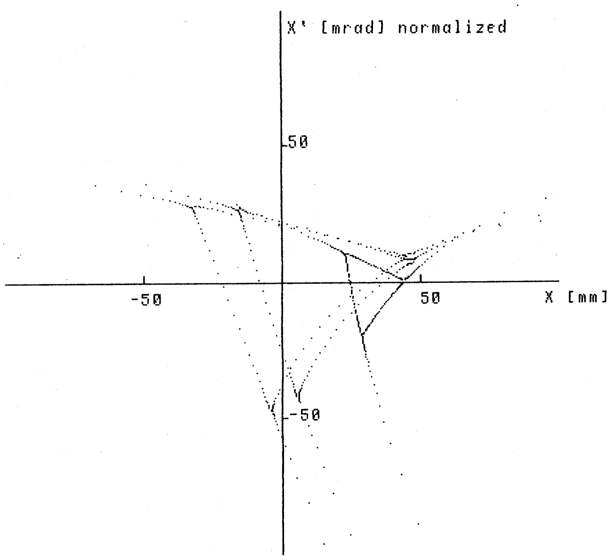

Fig. 3 The separatrices and outgoing trajectories of beams with $\Delta P/P0=$ 0. , 0.2 and 0.6 %, respectively, at ES(α=0)

Results of the simulation shows that the separatrix size is beyond two times of prediction by the theory. We have weakened S and Q0 a little to get the separatrix with r=50 mm. As a result, outgoing tracjectories can not be overlapped each other, as the strength S is not so weak but that separatrices and outgoing trajectories are curved. The stable fixed point is not always on the straight line $X'=\alpha X$. As shown in Fig. 3, at the entrance of ES, the outgoing trajectories have been made pass by a point (64 mm, 17 mrad), which corresponds to (64 mm, 1.7 mrad) on the unnormalized phase space, as near as possible. Only this requirement is sufficient for good beam extraction efficiency. Comparison is shown in Table 2 between the results of the computer simulation and theoretical predictions.

Table 2
Comparison between computer simulation and theory

	simulation		theory	
tune value Q0	1.6755		1.6822	
phase (rad)	0.31		0	
B"*L (kG/m)	92 (S=6.01)		119 (S=7.80)	
	dB1"L=dB4"L=37, dB2"L=-75, dB3"L=1			
chromaticity	-0.26		-0.91	
dP/P0 (%)	separatrix (mm*mrad)	separation (mm)	separatrix (mm*mrad)	separation (mm)
0	70 pai	7.6	105 pai	8.4
0.2	54	7.4	67	7.0
0.6	10	3.3	17	4.2
0.8	+0	3.5	4	2.8

Conclusions

Beam trace on computer simulation shows that the separatrix size is beyond two times of prediction by the theory. By decreasing strength S of the sextupole magnets and the tune value Q0, we have found the solution for the third order resonance extraction. The maximum separatrix area at the time of extraction is 54π mm*mrad for 1.3 GeV proton beam with $\Delta P/P=\pm 0.2$ %, which corresponds to 54π mm*mrad for 200 MeV/u beam with e/m=1/3. The emittance of the extracted beam is about 0.3π mm*mrad. The extraction efficiency exceeds 85 %.

REFERENCES

1. R. Cappi et al., CERN/PS/LEAR 82-3, 1982
2. K. Endo et al., IEEE Trans., NS-26, #3, p3170, 1979
3. M. Q. Barton, Proc. of the 8th Int. Conf. on High Energy Accelerators, p85, 1971
4. W. Hardt, PS/DL/LEAR 81-6, 1981