THE WAKE POTENTIAL AND THE ENERGY SPREAD OF A HIGH-CURRENT SINGLE BUNCH

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ABSTRACT

The Osaka University Single Bunch Electron Linear Accelerator has been increased the single bunch charge from 14 nC to 67 nC (4×10^{11} e) by means of an injector with three subharmonic-prebunchers. The energy spread of the single bunch depends on the phase-angle. The minimum energy spread is obtained by controlling the phase-angle and it depends on the single bunch charge. The energy spread of the single bunch in the range between 0 nC and 16 nC is estimated to be 1 %, and the spread increases up to 2.5 % in the range between 40 nC and 67 nC. The most minimized energy spread is obtained at 33 nC, and it is estimated to be 0.7 %. The wake potential is calculated both by the analytical method and by the TWA-program.

Introduction

Recently, there has been great interest in a high-current single bunch accelerated by an electron linear accelerator. The energies realistically attainable by an electron-positron storage ring are limited to several hundreds GeV by the power ballance between the external rf power and the power loss imposed by synchrotron radiation. The linear collider, therefore, seems to be one of the feasible accelerators to reach particle energies on the order of 1 x 1 TeV in an electron-positron colliding beam-machine. The luminosity for the linear collider is determined by the bunch parameters at the interaction point where two single bunches collide after passing through each of the final focusing system. The number of particles[1] ought to be 1×10^{11} (16 nC), in order to obtain the high-luminosity greater than 1×10^{32}. The single bunch charge, however, has been increased to the maximum charge limited by the wakefields due to the bunch-cavity interaction.

The wakefields generated by an electron in the single bunch give rise to the forces acting on the successive electrons in the single bunch. The transverse components of the wakefields deflect the electrons and increase the beam emittance, while the longitudinal components change both energies and current distribution of the single bunch itself.

If a unit point charge passes through an rf-structure, the wake potential $W(\tau)$ is defined as the potential experienced by a test particle following a distance $c\tau$ behind the unit charge. The amplitude of the wake potential depends on the frequency for a resonant mode in the rf-structure. The scaling[2] with frequency for the longitudinal wake and the transverse wake per unit of the structure is given by

$$W_L \text{ (longitudinal)} = \omega^2 , \tag{1}$$

$$W_{Td} \text{(transverse-dipole)} = \omega^3 , \tag{2}$$

$$W_{Tq} \text{(transverse-quadrapole)} = \omega^5 . \tag{3}$$

It has been reported that the instability due to the transverse components limits the single bunch charge in an S-band structure at SLC. As for an L-band structure, the transverse components might be neglected, while the longitudinal components is dominant, since both the dipole and the quadrapole transverse wake potentials clearly decrease with the rf-frequency.

Longitudinal Wakefields

The longitudinal wake potential is calculated both by the analytical model and by the computer code named TWA(Transient Wave Analysis). The longitudinal wake potential for the short time-range less than 50 ps is analytically expressed by

$$W(\tau) = A \left(\exp -(\tau/B)^n \right) , \tag{4}$$

where A = 226 V/pC/m, B = 6.13 ps and n = 0.605 for SLC structure.[2] Two parameters, A and B, are roughly proportional to ω^2 and ω^{-1} respectively, the wake potential for L-band structure is obtained by scalling the resonant frequency ω. The longitudinal wake potential for L-band structure can be calculated by using the parameters of A = 46.8 V/pC/m, B = 13.5 ps and n = 0.605. The longitudinal wake potentials per unit length for a point charge of 1 pC are shown in Fig. 1.

The wake potential $U_b(t)$ for a single bunch can be obtained by integration of the wakefields while the single bunch travels through the rf-structure and leaves it.

$$U_b(t) = \int_{-\infty}^{\infty} W(t - \tau)\, I(\tau)\, d \tag{5}$$

$$= \int_{-\infty}^{\infty} W(\tau)\, I(t - \tau)\, d \quad , \tag{6}$$

where $I(\tau)$ is a current distribution of a single bunch.

For a Gaussian bunch, Eq.(7) can be written in the form

$$U_b(t) = \frac{A\, c\, e\, N_b}{\sqrt{2\pi}\,\sigma_z} \exp\left[- \left(\frac{t - t'}{B} \right)^n \right] \exp\left[-t'^2 c^2 / 2\sigma_z^2 \right] dt' . \tag{7}$$

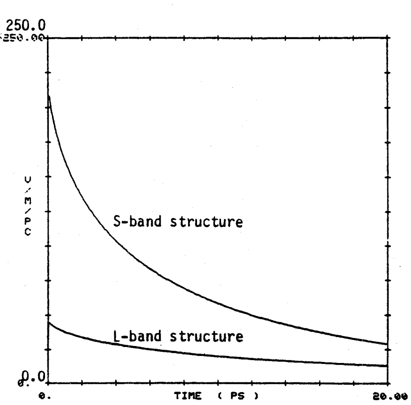

Fig. 1. The longitudinal wake potentials per unit length for a point charge of 1 pC.

The wake potentials of a single bunch of particles with $N_b = 1.0 \times 10^{11}$ are shown in Figures 2-a to 2-d for several values of the bunch length σ_z. There results show that the wake potential for L-band structure is smaller than the potential for S-band structure. It seems that the L-band structure is suitable for accelerating a high-current single bunch.

Loss Parameter

The total energy loss ΔU is expressed in terms of the wake potential $U_b(t)$ and the current distribution $I(t)$,

$$\Delta U = \int_{-\infty}^{\infty} U_b(t)\, I(t)\, dt\ . \tag{8}$$

The energy left behind in the rf-structure by the single bunch is equal to the total energy loss of the single bunch. Thus, the loss parameter k is given by

$$k = \frac{\int_{-\infty}^{\infty} V_b(t)\, I(t)\, dt}{q^2}\ , \tag{9}$$

where q is the single bunch charge. The loss parameter k for several values of the bunch length σ_z are shown in Table 1.

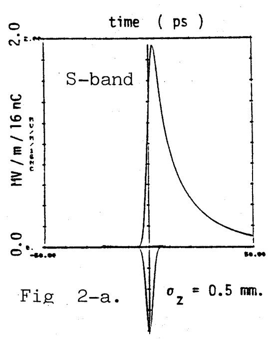

Fig 2-a. σ_z = 0.5 mm.

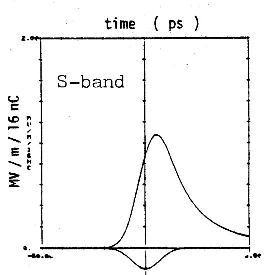

Fig. 2-b. σ_z = 2.0 mm.

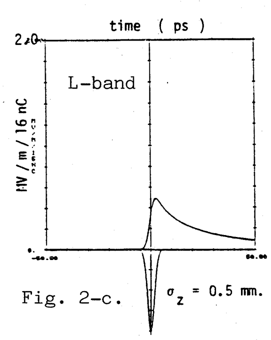

Fig. 2-c. σ_z = 0.5 mm.

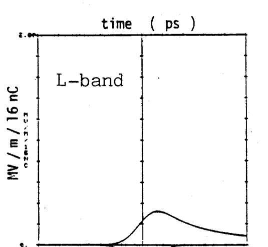

Fig. 2-d. σ_z = 2.0 mm.

Fig. 2-a, b, c and d. The wake potential $U_b(t)$ of a single bunch of particles with $N_b = 1.0 \times 10^{11}$.

Bunch Length σ_z (mm)	Loss parameter k (MeV/m/1x10^{11}particles)	
	S-band structure	L-band structure
0.5	1.209	0.291
1.0	0.976	0.253
2.0	0.728	0.207
4.0	0.492	0.157

Table 1. The loss parameter k for several values of the bunch length.

Energy Spread of the Single Bunch

When a single bunch is accelerated by a linear accelerator, the total energy gain of an electron at time, t, can be obtained by adding the wake potential to the external accelerating voltage,

$$E(t) = E_0 \cos(\omega t - \theta) + U_b(t)\ , \tag{10}$$

where θ is the phase-angle between the single bunch and the accelerating voltage. With increase of bunch charge, the wake potential $U_b(t)$ increases from the small fraction of the accelerating voltage to the value which is large enough to distort the net accelerating voltage. Figure 3-a to 3-d show the results of calculation of the net accelerating voltage for the single bunch of 32 nC in the L-band structure of Osaka University linac. The bunch shape is assumed to be a gamma function the parameter of which are determined by the experimental data.

When a low-current single bunch is accelerated by the linear accelerator, the distortion of the accelerating field will be negligible. Figure 4-a shows the phase dependence of the energy spectrum for the single bunch of 0.5 nC.[3] By increasing rf-phase, the head of the bunch approaches the crest of the accelerating wave, and the maximum energy increases until the bunch-head reaches the crest. When the bunch-head leads the crest, the maximum energy is kept constant, since some electrons are accelerated on the crest of the accelerating wave. When the bunch-tail leads the crest, the maximum energy decreases with the phase.

Figure 4-b shows the dependence of the energy spectrum on the rf-phase for the single bunch of 45 nC.[4] With the increase of rf-phase, the maximum energy increases, since the head of the bunch approaches the crest. When the bunch-head leads the crest, the maximum energy decreases by the wake potential. As the bunch-tail approaches the crest, the maximum energy re-increases. Through the experiments, the energy spectrum of the single bunch has been obtained for the bunch charge from 0.5 nC to 67 nC. These experimental data show that the average energy loss of the single bunch due to the wake potential is estimated to be 0.29 - 0.30 MeV/m/16 nC, while the depth of the wake potential can be estimated to be about 0.4 MeV/m/16 nC. The wake potential of the single bunch reaches to 10 % of the external accelerating energy.

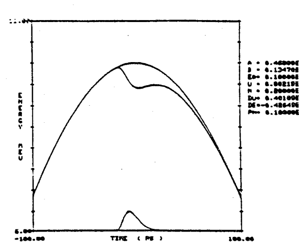

Fig. 3-a. Phase θ = 3.45°.

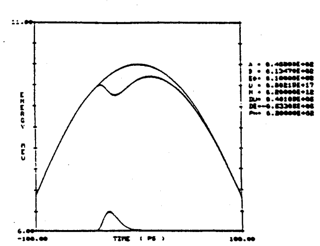

Fig 3-b. Phase θ = 13.45°.

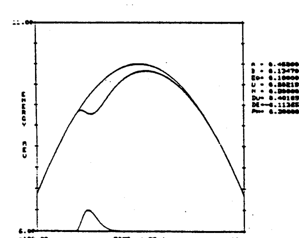

Fig. 3-c. Phase θ = 23.45°.

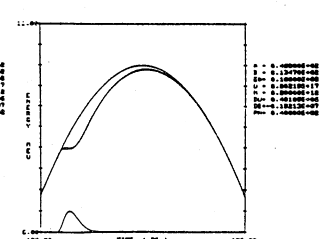

Fig 3-d. Phase θ = 34.45°.

Fig. 3-a, b, c and d. Results of calculation of the net accelerating voltage in an L-band structure for several values of the phase-angle. The number of particles, $N_b = 2.0 \times 10^{11}$ (32 nC).

Minimum Energy Spread of the Single Bunch

The minimum energy spread can be obtained, when the single bunch is accelerated at the positive phase-angle where the negative going slope of the accelerating voltage waveform is made to cancel with the positive going slope of the wake potential. The effect of the cancel depends on the shape of the wake potential which is determined not only by the single bunch charge but also by the shape of the single bunch.

Figure 5 shows the minimum energy spread which can be obtained by controlling the phase-angle to accelerate the single bunch by the Osaka Univ. linac. The energy spread is observed to be 1.0 % for the single bunch charge in the range 0 - 16 nC, in spite of the increase in the energy spread with the increase of the bunch charge. It seems that the increase in the energy spread due to the wake potential is cancelled by the phase-controll. For the bunch charge within the region 16 - 40 nC, the decrease in the energy spread is effective by the cancel of the negative going slope with the wake potential. The minimum spread is observed to be 0.7 % at the single bunch charge of 33 nC. When the single bunch of high-current greater than 40 nC is accelerated, the energy spread increases with the single bunch charge. It seems that the increase in the single bunch charge gives rise to the increase in the going slope of the wake potential, and it exceeds the slope of the external accelerating voltage. If the energy spread of the single bunch greater than 33 nC is to be minimized, the higher gradient of external accelerating voltage or the longer length of the single bunch is required.

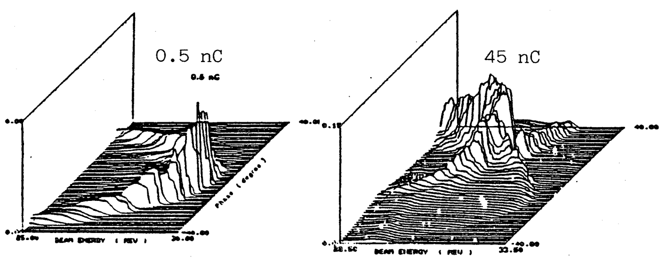

Fig. 4-a and 4-b. The energy spectrum of the single bunch.

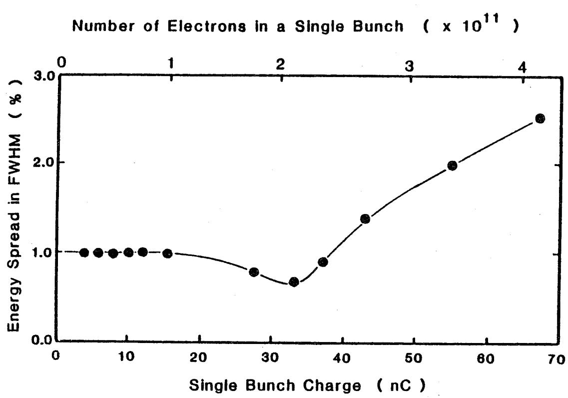

Fig. 5. The minimum energy spread for the bunch charge.

Time-Domain Analysis of the Wake Potential

The wakefields excited by a single bunch passing through an rf-structure can be directly computed in time-domain without any assumptions or restrictions required for the idealized cases. The BCI-program has been used to calculate the wakefields in a cavity, especially for the bunches with relatively long bunch-length compared with the cavity size. When the time-domain method is applied to the wakefields generated by a single bunch which is accelerated in a periodic structure of a linear accelerator, the large size of memories over the limited value of the computer is required. It is because the mesh size should be smaller than the bunch-length, which is typically shorter than 2 mm in a linear accelerator. The wake potential of the single bunch in a unit cell of the L-band linear accelerator has been calculated by using the time-domain program J-BCI.

TWA-program posesses an advantage of smaller memory size than any other time-domain programs, since it treats only one field parameter, the wave potential. Figure 6 shows the induced fields in a disk-loaded structure of L-band linac. When the single bunch passes through the disk, the wakefields are radiated at the inner hole of the disk, and they propagate in a structure. It shows that the short range force of the wakefields is on the point of overtaking the rear of the bunch-tail. Figure 7 shows the wake potential of the single bunch passed through the L-band periodic structure.

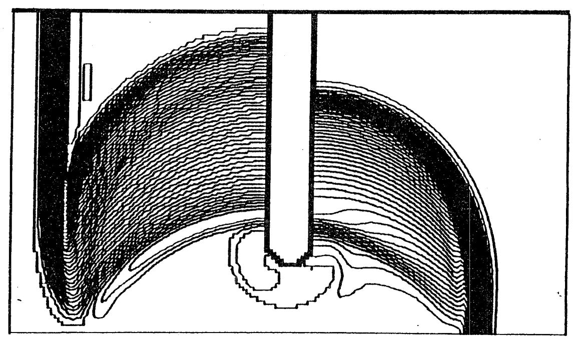

Fig. 6. Wakefields generated by a single bunch passing through a disk-loaded structure. The bunch length is 2 mm long, while the thickness of the disk is 13 mm.

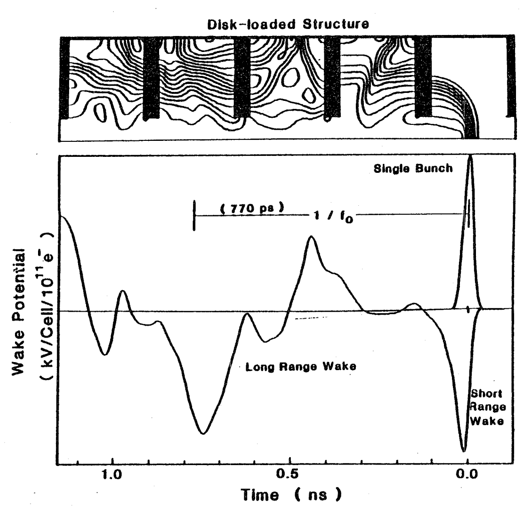

Fig. 7. The wake potential of the single bunch passing through an L-band periodic structure. The bunch length is 4 mm.

References

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4) S. Takeda, N. Kimura, K. Tsumori and M. Kawanishi, Proc. 9th Meet. on Linear Acc. (1984) 103.