LONGITUDINAL STABILITY LIMIT FOR ELECTRON BUNCHES IN A DOUBLE RF SYSTEM

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Abstract

Combination of main and higher harmonic RF systems, which produces a zero slope of the RF waveform, realizes a biquadratic RF potential in the synchrotron phase space and yields a longer bunch and a very large synchrotron frequency spread. In this paper, a dispersion relation is presented for electron bunches in the biquadratic RF potential by using the canonical formulation of Suzuki. A greatly increased stability limit due to Landau damping is shown numerically.

HAMILTONIAN AND DISPERSION RELATION

A double RF system is considered as a useful cure for bunched beam instabilities and synchro-betatron resonances, because it permits the control of a bunch length and phase oscillation frequency.[1] To maximize the bunch length, RF waveform should have zero slope at the bunch centre. This condition gives a biquadratic RF potential and Hamiltonian can be written as[2]

$$H = -\frac{eV_0\cos\phi_s}{4\pi\omega_{rf}}\left[\frac{\phi'^2}{Q_{so}^2} + \frac{n^2-1}{12}\phi^4\right] = -\frac{eV_0\cos\phi_s}{4\pi\omega_{rf}}\frac{n^2-1}{12}\phi_{max}^4 , \qquad (1)$$

where the prime denotes the differentiation with respect to angular position θ[3]; $\phi' = d\phi/d\theta$. The notations and their numerical values used in a later example are summarized in Table 1. From eq. (1), the phase motion ϕ, ϕ'/Q_{so} can be expressed by the Jacobian elliptic function cn(u) and the product sn(u)dn(u). The action angle variables I, ψ are given by

$$I = -\frac{eV_0\cos\phi_s}{4\pi^2\omega_{rf}Q_{so}}\frac{4\sqrt{2}}{3}K\left(\frac{1}{\sqrt{2}}\right)\sqrt{\frac{n^2-1}{12}}\phi_{max}^3 , \qquad (2)$$

$$\psi = \Omega_0\frac{\pi}{2K(1/\sqrt{2})}\sqrt{\frac{n^2-1}{6}}\phi_{max}t \equiv \Omega_s t . \qquad (3)$$

We assume that a longitudinal solution of the Vlasov equation can be described by the sum of the stationary distribution function f_0 which is a function of only I[3] and a perturbed distribution function $R_m(I)$

$$f = f_0(I) + R_m(I)\exp\left(-\frac{\Omega}{\omega_0}\theta + i\,m\,\psi\right) . \qquad (4)$$

The self-consistent solution $R_m(I)$ can be obtained by solving the eigenvalue equations yielded by expansion with orthogonal polynomials.[4] However, here, instead of this method, we use the synthetic kernel approximation[5] which neglects the higher radial modes except the lowest radial mode, because it gives a dominant term in general. After some algebla, we obtain a following dispersion relation for coherent frequency Ω

$$1 = KM\int\frac{z^{2m+3}\exp(-0.1142z^4)}{\Omega - m\Omega_s}e_m^2\,dz , \qquad (5)$$

where

$$M = i\sum_{p=-\infty}^{\infty}\frac{Z_L(p\omega_0+\Omega)}{p\omega_0+\Omega}C_m^2\left(\frac{p\omega_0+\Omega}{h\omega_0}\sigma_\phi\right) , \qquad (6)$$

$$C_m\left(\frac{p}{h}\sigma_\phi\right) = \int_0^\infty z^{m+3}\exp(-0.1142z^4)e_m J_m\left(\frac{p}{h}\sigma_\phi z\right)dz , \qquad (7)$$

$$K = \frac{m\omega_0^2 I_b h}{\sigma_\phi\frac{\sigma_p}{p}E/e}\times 0.0488 , \qquad (8)$$

and e_m is the normalization constant defined by relation

$$\int_0^{\infty} \exp(-\ 0.1142z^4)\ z^{2m+3}\ e_m^2\ dz\ =\ 1 \quad . \tag{9}$$

STABILITY LIMIT

We are interested in growing solutions; $\Omega = \mathrm{Re}\Omega + i\mathrm{Im}\Omega(\mathrm{Im}\Omega > 0)$. In order to make the effect of the large synchrotron frequency spread on Landau damping clearly, we shall calculate a threshold current for the given impedance whose parameters are shown in Table 1. The growth rate $\mathrm{Im}\Omega$ for the dipole mode $m = 1$ is shown in Fig. 1 by the solid line. It is found that the clear threshold appears under which the beam is stable. The broken line in Fig. 1 shows the growth rate for the case of no synchrotron frequency spread in the single RF system. Taking into account the real part of the frequency shift, we obtain a rough stability criterion that the absolute value of the coherent frequency shift in the single RF system should be smaller than the spread in the synchrotron frequency between center and rms of the bunch in the double RF system.

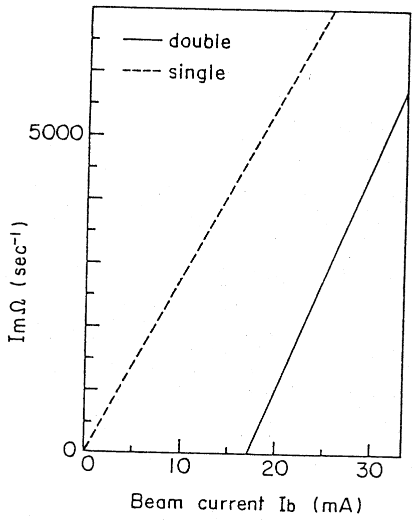

Fig. 1 The growth rate $\mathrm{Im}\Omega$ for the dipole mode $m = 1$ for the case of the narrowband impedance.

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REFERENCES

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2) Y. Chin: KEK Report 82- (1982).
3) T. Suzuki: KEK Report 82-10 (1982) and also submitted to Particle Accelerators.
4) Y. Chin, K. Yokoya and K. Satoh: KEK Preprint 82-18 (1982) and also submitted to Particle Accelerators.
5) F.J. Sacherer: CERN/SI-BR/72-5 (1972).

Table 1

Notations and numerical values used for a example of a stability limit

V_0	=	peak voltage of the main RF	
ϕ_s	=	stable phase angle of the main RF	
ω_{rf}	=	$h\omega_0$ = harmonic angular frequency	500 MHz × 2π
ω_0	=	revolution angular frequency	100 kHz × 2π
Q_{so}	=	Ω_0/ω_0 = synchrotron tune in the single RF system	0.1
n	=	ratio of main RF frequency and higher harmonic RF frequency	2
e	=	elementary charge	
m	=	azimuthal mode number	
I_b	=	beam current	
E	=	beam energy	8 GeV
α	=	momentum compaction factor	10^{-3}
σ_ϕ	=	rms bunch length measured in RF phase angle	0.447
σ_p/p	=	rms momentum spread of the bunch center	4.19×10^{-3}
f_r	=	resonant frequency	1000.015 MHz
R_s	=	shunt impedance	1 GΩ
Q	=	quality factor	80,000