THE EFFECT OF A RESONANCE ON THE BEAM IN THE RIKEN SSC

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Recently, measurements were done for magnetic fields in the system of two full-scale sector magnets, and some properties of the RIKEN separated sector cyclotron have been recalculated by using the measured data. As one of them, focusing frequencies for typical particles and energies were also examined, and it was confirmed that the results were not so different from those which had obtained using magnetic fields in model magnets. According to the calculation, a 9 MeV proton beam crosses the resonance line $\nu_r - 2\nu_z = 0$ in a (ν_r, ν_z) space during acceleration. The focusing frequency is shown in Fig. 1. The resonance is known to be a quadratic non-linear coupling resonance and to be driven by the existance of the second-order derivatives of the magnetic induction. The beam encounters the resonance at the energy of around 144 MeV (R = 295 cm).

We have only data of the axial component of the magnetic induction $B_z(r, \theta)$ on the median plane, that is $B_z(r, \theta) = B(r, \theta, z = 0)$. The field $B(r, \theta, z)$ off the median plane is obtained in a power expansion in z using symmetry with respect to the plane.[1)]

$$B_z = B + \frac{1}{2} z^2 f^2,$$

where $$f_2 = -\left(\frac{\partial^2 B}{\partial r^2} + \frac{1}{r}\frac{\partial B}{\partial r} + \frac{1}{r^2}\frac{\partial^2 B}{\partial \theta^2}\right)$$

From rot B = 0, radial and azimuthal components of the magnetic field are given by

$$B_r = z\frac{\partial B}{\partial r} + \frac{1}{6} z^3 . f_3$$

$$r\cdot B_\theta = z\frac{\partial B}{\partial \theta} + \frac{1}{6} z^3 . g_3 ,$$

where $$f_3 = \frac{\partial}{\partial r} f_2 = -\left(\frac{\partial^3 B}{\partial r^3} + \frac{1}{r}\frac{\partial^2 B}{\partial r^2} - \frac{1}{r^2}\frac{\partial B}{\partial r} + \frac{1}{r^2}\frac{\partial^3 B}{\partial r \partial \theta^2} - \frac{2}{r^3}\frac{\partial^2 B}{\partial \theta^2}\right)$$

and $$g_3 = \frac{\partial}{\partial \theta} f_2 = -\left(\frac{\partial^3 B}{\partial r^2 \partial \theta} + \frac{1}{r}\frac{\partial^2 B}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\partial^3 B}{\partial \theta^3}\right)$$

Calculations have been done modifying an existing program ACCELP.[2)] Figures 2a·b show the radial and axial oscillations of particles with respect to the central orbit during acceleration. In order to examine the effect of the higher-order terms on amplitudes, two particles of the initial amplitude of 5 mm in each direction were traced in use of equations without (a) and with (b) the terms. We cannot find the remarkable difference in amplitude between them. These initial amplitudes correspond to the emittances of around 30 and 20 mm·mrad, respectively, and these are probable values in practice. Thus the influence of the resonance on amplitudes is practically negligibly small. It is considered to be due to a large energy gain per turn. Further study is in progress.

References

1) J.P. Blaser and H.A. Willax, IEEE Trans. NS-13 pp 194.
2) N. Nakanishi, Reports I.P.C.R. (in Japanese) 57. 189 (1981)

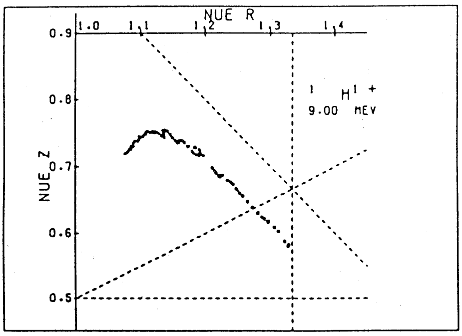

Fig. 1. Focusing frequency of a 9 MeV proton beam.

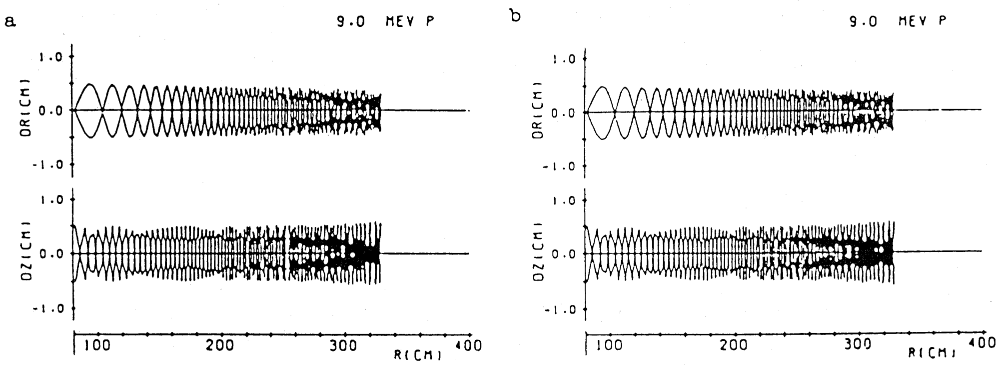

Fig. 2. Calculated oscillation patterns around the central orbit during acceleration. Calculations were done without (a) and with (b) higher-order terms.