MAGNETIC FIELD AND BEHAVIOURS OF THE PROTON BEAM OF THE BOOSTER SYNCHROTRON

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Magnetic field properties such as the excitation dependences and the field distributions of the field strength, field gradient, multipole fields and the effective length[1] are analyzed and accuracy of the estimated tune, tune shift and chromaticity are discussed. Lattice parameters such as tune, twiss parameters and dispersion functions are calculated by using SYNCH program,[2] where normal entrance and square cut fringing approximation are assumed.

Figure 1 shows the tune diagram calculated with the data of magnetic measurement and that observed with the accelerated proton beam.[3] The discrepancy between them might be explained by the error of the field measurements, the accuracy of the approximation adopted in the program, space charge effect etc.. The error of the horizontal and vertical tune δQ_x and δQ_z due to the estimation error of the field gradient and the effective length can be written,

$$\delta Q_x \simeq 3.14\ \delta k_F/k_{F0} - 1.67\ \delta k_0/k_{D0} + 3.87\ \delta \ell_G^F/\ell_{G0} - 0.75\ \delta \ell_G^D/\ell_{G0}$$
$$-2.65\ \delta \ell_B/\ell_{B0} + 2.55\ \delta \ell_T + 0.09\ \delta {\ell_T}^2 + 0.81\ \delta \ell_E\ , \quad (1)$$

$$\delta Q_z \simeq -2.64\ \delta k_F/k_{F0} + 6.01\ \delta k_D/k_{D0} - 1.71\ \delta \ell_G^F/\ell_{G0} + 7.12\ \delta \ell_G^D/\ell_{G0}$$
$$-4.38\ \delta \ell_B/\ell_{B0} + 4.51\ \delta \ell_T - 2.03\ \delta {\ell_T}^2 + 0.84\ \delta \ell_E\ , \quad \ell \text{ in meter}, \quad (2)$$

where δ denotes the error of the estimation, k the field gradient, ℓ the effective bending and gradient length, subscript T transition sector between F and D sector, and E the end sector, respectively. The tune is designed to be Q_x = 2.20 and Q_z = 2.30. The discrepancy between them at medium field can be explained by the error of the field gradient and the effective length; underestimation of the field gradient and the effective length and over-estimation of the effective bending length by 0.2 % give the horizontal tune shift of about 0.02. The discrepancy at high field can be explained by setting error of the field level during the measurement. Setting error of the field level by 4 % gives the tune error $\delta Q_x \simeq -0.02$ and $\delta Q_z \simeq 0.01$ at high field. Setting error of 4 %, however, is too large compared with that of our expectation.

The chromaticity due to the sextupole fields can be separated into the chromaticity ξ_0 and ξ_{fringe} which are contributions from the normal and fringing field;

$$\xi \equiv \frac{\delta Q}{\frac{\delta p}{p}} = \xi_0 + \xi_{fringe} \quad (3)$$

$$\xi_x \simeq \{0.970 + 0.884\ k_{2F}\ell_G^F + 0.320\ k_{2D}\ell_G^D\}$$
$$+ \{0.675\ k_{1F}\left(\frac{\partial \ell_G}{\partial x}\right)^{FT} + 0.565\ k_{1D}\left(\frac{\partial \ell_G}{\partial x}\right)^{DT} + 1.022\ k_{1F}\left(\frac{\partial \ell_G}{\partial x}\right)^{FE}\} \quad (4)$$

$$\xi_z \simeq -\{0.870 + 1.410\, k_{2D}\ell_G^D + 0.900\, k_{2F}\ell_G^F\}$$

$$-\{1.080\, k_{1F}(\frac{\partial \ell_G}{\partial x})^{FT} + 1.160\, k_{1D}(\frac{\partial \ell_G}{\partial x})^{DT} + 0.588\, k_{1F}(\frac{\partial \ell_G}{\partial x})^{E}\} \qquad (5)$$

where superscripts FT is transition sector of F sector's side and DT at transition sector of D sector's side. Figure 2 gives the excitation dependence of the chromaticity. The horizontal chromaticity is observed with the accelerated proton beam. Agreements of the chromaticity between them is satisfactory. In above equations, typical parameters are:

$$k_{2F}\ell_G^F \simeq -0.804\ m^{-1},\ k_{2D}\ell_G^D \simeq -0.93\ m^{-1},\ k_{1F}(\frac{\partial \ell_G}{\partial x}) \simeq k_{1D}(\frac{\partial \ell_G}{\partial x})^{DT} \simeq 4.2\ m^{-1}$$

and $k_{1F}(\frac{\partial \ell_G}{\partial x})^{FE} \simeq -5.0\ m^{-1}$.

In the horizontal chromaticity we see fringing field at T sector cancels with that at E sector. In the vertical chromaticity, however, this cancelling effect is not enough. Vertical chromaticity is estimated to be -6.8 for medium field. This value is larger than that obtained from the knockout method by a factor of several.

The tune spread caused by the octupole can be written after similar calculation as above;

$$\delta Q_x \simeq -11(\frac{\delta p}{p})^2 \qquad \text{and} \qquad \delta Q_z \simeq 17.5(\frac{\delta p}{p})^2.$$

Multipole fields also produce the amplitude dependent tune shift. Typical values of the emittance $E_x = 56$ mm mrad and $E_z = 14$ mm mrad give the tune shift of $\delta Q_x = -0.6 \times 10^{-4}$ and $\delta Q_z = 4.6 \times 10^{-4}$ at injection, $\delta Q_x = -1.3 \times 10^{-4}$ and $\delta Q_z = 4.8 \times 10^{-4}$ at medium field and $\delta Q_x = -1.7 \times 10^{-4}$ and $\delta Q_z = 4.2 \times 10^{-4}$ at ejection field level.

References

1) M. Kumada, H. Sasaki, K. Takikawa, H. Someya, T. Kurosawa and Y. Miyahara: "The magnetic field measurement of the Booster synchrotron magnet" to be published in KEK report.
2) A.A. Garren and J.W. Ensebio: "SYNCH-A Computer System for Synchrotron Design and Orbit Analysis", UICD-10153, 1965.
3) Y. Miyahara, H. Sasaki, H. Someya and K. Muto: "Measurement of the betatron oscillation frequency by the RF knock-out method", KEK-76-16.

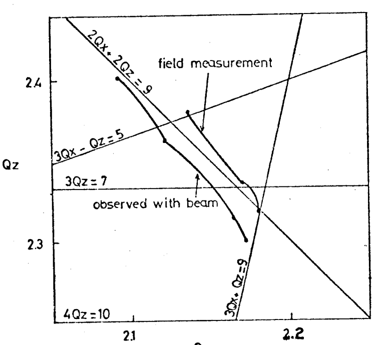

Fig.1 Tune diagram

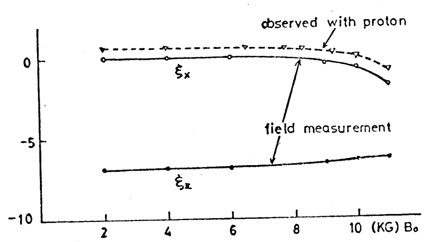

Fig.2 Chromaticity