REMARKS ON DESIGNING PREINJECTOR ELECTRODES BY THE AID OF A DIGITAL COMPUTER

KEK S. Inagaki, K. Ebihara and S. Okumura

We have modified the program reported before[1] to compute the potential and trajectories in a electrostatic field and incorporated a plotter subroutine to visualize them. We used HITAC-8700 computer and WATANABE SOKKI WX-535 X-Y plotter.

C.R. Emigh[2] analyzed the axial symmetrical field to form a laminar flow of a constant radius r_0, and showed that the potential V(Z,R) should be given by

$$\frac{V(Z,R)}{\alpha} = Z^{4/3} + \int_0^\infty \frac{4}{9\Gamma(2/3)} \cdot \frac{1}{k^{2/3}} \left\{ U(R) - 1 \right\}^{-kZ} dk$$

$$U(R) = \frac{\pi k}{2} \left\{ J_1(k) Y_0(kR) - Y_1(k) J_0(kR) \right\}$$

for $R \equiv r/r_0 \geq 1$ $\quad Z = z/r_0 \geq 0$,

where α is given by

$$\alpha = \left[\frac{9j}{4\varepsilon_0} \left(\frac{m}{2e} \right)^{1/2} \right]^{2/3}$$

and k is an arbitrary variable.

The multi-gap acceleration is effective to maintain the adequate field and to hold out a high electric field. However, the number of gaps is limited to a finite number and the above expression is normalyzed by α. Our aim is to calculate the potentials given by any Dirichlet boundary conditions and trajectories for any currents and particles, taking the space charge effect into consideration.

Our calculation is divided into three steps; Poisson equation, equation of motion and space charge force, which are expressed by

$$\frac{\partial^2 U(z,r)}{\partial r^2} + \frac{1}{r}\frac{\partial U(z,r)}{\partial r} + \frac{\partial^2 U(z,r)}{\partial z^2} = -\frac{\rho(z,r)}{\varepsilon_0} \quad (\text{for } r \neq 0),$$

$$2\frac{\partial^2 U(z,r)}{\partial r^2} + \frac{\partial^2 U(z,r)}{\partial z^2} = -\frac{\rho(z,r)}{\varepsilon_0} \quad (\text{for } r = 0),$$

$$r = r_0 + \left(\frac{dr}{dz}\right)_0 (z - z_0) + \frac{1 + \left(\frac{dr}{dz}\right)^2}{4V(z_0, r_0)} \left[\frac{\partial V}{\partial r} - \left(\frac{\partial V}{\partial z}\right)\left(\frac{dz}{dr}\right) \right] \cdot (z - z_0)^2 ,$$

$$\rho(z,r) = \sqrt{\frac{m}{2eV}}\, i_0 \frac{S_0}{S} \cdot \frac{\Delta \ell}{\Delta z} ,$$

where the notation should be referred to Ref.1. We applied this for the KEK proto-type preinjector electrodes,[3] which are shown in Fig.1. The equipotential surfaces without beam and with a 300 mA proton beam of zero emittance at the plasma boundary were obtained as shown in Fig.2 and Fig.3, respectively. In executing the same kind of computations, the followings should be remembered.

1. Boundary condition in case the boundary is not located on the mesh points. We got the same results in the accuracy of 0.1 % with the two different methods in introducing the boundary condition. (i) To give the boundary only by a distribution of the mesh points. (ii) To express the boundary condition by geometrical relations between the circumferential mesh points and the boundary lines. For the latter case, the boundary condition claims the memories as large as four time of the number of mesh points. The boundary conditions contain the error of order h for the second order differential equations,[4] where h is a size of mesh. If we use the same size of computer memories, we recommend to use finer mesh, which guarantees the better convergence of the order of h^2.[5]

2. Division of Region. If we divide the whole region into several smaller regions and use the different mesh sizes, the attention has to be payed to the estimation of $|U^{(4)}(x)|$, since the discretization error is proportional to $M_4 h^2$, where $M_4 = \max|U^{(4)}(x)|$. The error becomes maximum at the points where the distances from the boundary is largest.

3. Optimum Acceleration Factor. In spite of the use of well-known Carré's method,[6] the convergence is sometimes unsatisfactory. If it is clear that $\chi^{(n)}$ is far depart from the real value χ, it is recommended to use $\omega \to 2\text{-}0$. If $\chi^{(n)}$ oscillates around same values, $\omega = 1$ should be used.

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References

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electrode	applied voltage	inner diameter	distances from the plasma boundary	gradicnt of the electrode
beam forming	800 kV	25.0 Φ	0.0 mm	2.41 (67.5°)
extraction	750	40.0	22.5	3.30 (73.2°)
first accelerating	600	45.0	63.6	5.00 (78.7°)
second accelerating	400	50.0	107.0	8.65 (83.4°)
third accelerating	200	55.0	145.0	9.20 (83.8°)
earthed	0	60.0	180.0	11.00 (84.8°)

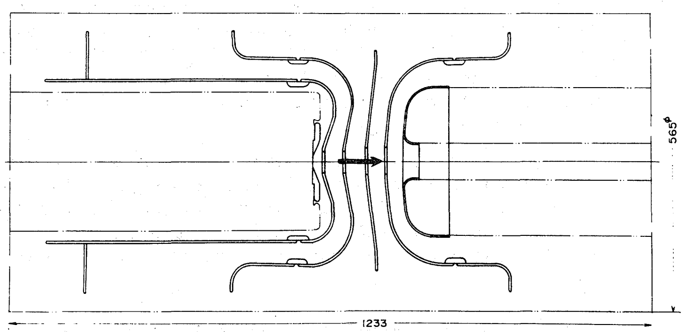

Fig.1.

KEK proto-type preinjector electrodes.

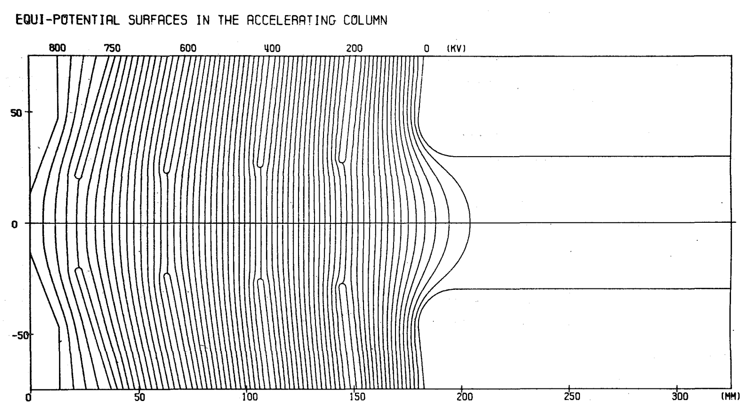

Fig.2.

Equi-Potential Surfaces without Beam. Their width is 12.5 (kV).

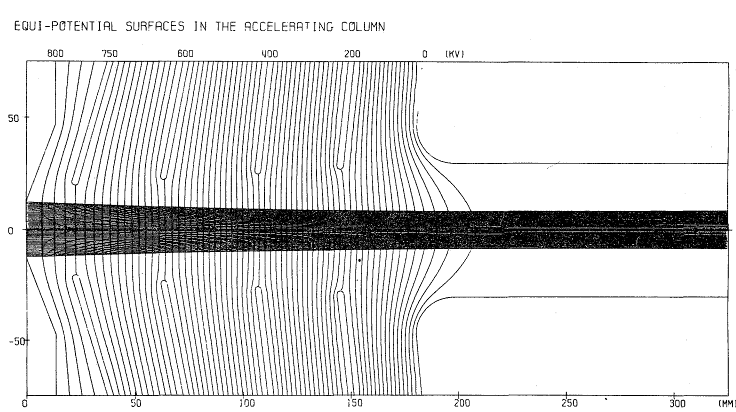

Fig.3.

Equi-Potential Surfaces with a 300 mA beam and trajectories. Their width is 12.5 (kV).