Proceedings of the 18th Linear Accelerator Meeting in Japan, Tsukuba, 21-23 July 1993

BEHAVIOR OF OFF-CENTERED BEAMS IN AN RFQ DUE TO IMAGE CHARGE FORCE

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ABSTRACT

Behavior of misaligned beams in an RFQ was analyzed by using a 3-D PIC (Particle-In-Cell) computer code QLASSI2. Trajectories of 2^8 macro particles were simultaneously traced using a supercomputer. The potential for the image charge force from the metallic walls was expanded into a Fourier-Bessel series including odd-number order harmonics. Results obtained with and without image charge effects have been compared with each other. Both calculations predict fairly similar behavior. There is, however, an observable difference not only on transmission, but also on trajectory and output emittance of the beam. It has been found that the amplitudes of coherent transverse beam oscillation in the RFQ are enhanced owing to the effects of image charge forces.

INTRODUCTION

In a beam-dynamics analysis of an radio-frequency quadrupole (RFQ) accelerator, the effects of image charge and space charge on the motion of beams are considered to be important for high current operation. In general, due to the image charge force, particles are attracted and deflected out of the beam. If the effects of image charge force are taken into account, the beam transmission can be decreased(1),(2). Since in practice the injection into an RFQ is not always ideally centered, the behavior of misaligned beams is being studied using QLASSI2 code.

The QLASSI2 ("Quadrupole Linear Acceleration Simulator with Space- and Image charge effect, version 2") — a modified version of QLASSI(3),(4), is a 3-D PIC (Particle-In-Cell) beam tracking computer code, in which not only the space charge effect, but also the image charge force for off-centered beams can be treated. In this code, the dynamics of the RFQ beam is calculated by integration of the equation of the particle's motion using fourth-order Runge-Kutta method.

CALCULATION OF 3-D IMAGE CHARGE POTENTIAL

In order to simulate the motion of the off-centered beams, a number of modifications have been added to the QLASSI code.

The electric potential in the quadrupole channel can be expressed as(3),(4)

$$U = U_{RFQ} + U_{sc} + U_{ic}, \qquad (1)$$

where U_{RFQ},U_{sc} and U_{ic} represent the general RFQ external, the space charge and the image charge potential, respectively.

For any test particle the total space charge potential U_{sc} is given by the sum of the Coulomb potential of all other particles in the bunch and its two nearest neighbor bunches $^{(4)}$.

With cylindrical coordinates, the image charge potential $U_{ic}(r,\varphi,z)$ can be expanded as a Fourier series:

$$U_{ic}(r,\varphi,z) = A_0(r,\varphi) + \sum_{n=1}^{\infty}\Big[A_n(r,\varphi)\cos nkz + B_n(r,\varphi)\sin nkz\Big]. \qquad (2)$$

Inside of the quadrupole channel Eq.(2) satisfies the Laplace equation:

$$\nabla^2 U_{ic} = 0. \qquad (3)$$

For simplicity of calculation a misaligned elliptic beam is assumed to be injected into the RFQ off-centered along the x-axis. Then the potential function has the symmetrical property:

$$U_{ic}(r,\varphi,z) = U_{ic}(r,-\varphi,z).$$

Under this condition the solution of Eq.(3) can be written as:

$$U_{ic}(r,\varphi,z) = \sum_{m=0}^{\infty} A_m\left(\frac{r}{r_0}\right)^m \cos m\varphi + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} I_m(nkr)\cos m\varphi \times\Big[A_{nm}\cos nkz + B_{nm}\sin nkz\Big], \qquad (4)$$

$$k = \frac{\pi}{L},$$

where L denotes the cell length, $I_m(nkr)$ the first kind of modified Bessel function and r_0 the quadrupole bore radius. From the boundary condition ($U_{sc} + U_{ic} = 0$ on the metallic vane surface), it is possible to determine the coefficients A_m, A_{nm} and B_{nm}. Then the image charge potential can be estimated.

Balancing the accuracy of calculted results and required CPU time, multipole moments up to order m=6 and n=3 have been proven to be appropriate. There are 49 terms of harmonics in the image charge potential in Eq.(4) including odd-number order ones.

RESULTS AND DISCUSSION

The behavior of misaligned beams in an 80-MHz four-vane RFQ$^{(5)}$ was analyzed. Table 1 shows main parameters for the calculation.

Table 1
MAIN PARAMETERS FOR BEAM DYNAMICS ANALYSIS

charge-to-mass ratio	1/16
operating frequency(MHz)	80
input energy(keV/amu)	5
normalized emittance(cm mrad)	0.05
total number of cell	273
characteristic bore radius,r_0(cm)	0.49
intervane voltage(kV)	78.9
injected beam current(mA)	10

The motion of the injected beam composed of 2^8 macro particles was calculated by using a CDC-ETA10 supercomputer. The elapsed time was about 27 seconds per acceleration cell. The general RFQ external potential U_{RFQ} used here is the linear two-term potential$^{(6)}$.

Table 2 gives the beam transmission percentage of beams injected at different off-centered points. Compared with the calculation without image charge force, the transmission in the run with image charge force was slightly decreased. The relative degradations of the transmission caused by the image charge force were almost the same in aligned and misaligned beams. In order to investigate the process of transmission degradation, the beam particle loss profile was calculated as shown in Fig.1. When taking the

Table 2
BEAM TRANSMISSION PERCENTAGES AT DIFFERENT CONDITIONS

misalignment (mm)	transimssion (%)	image charge force
0	67.6 68.4	ON OFF
0.5	66.0 66.8	ON OFF
1.0	60.9 60.9	ON OFF
1.5	53.4 53.9	ON OFF
2.0	44.1 45.3	ON OFF
2.5	37.1 39.1	ON OFF

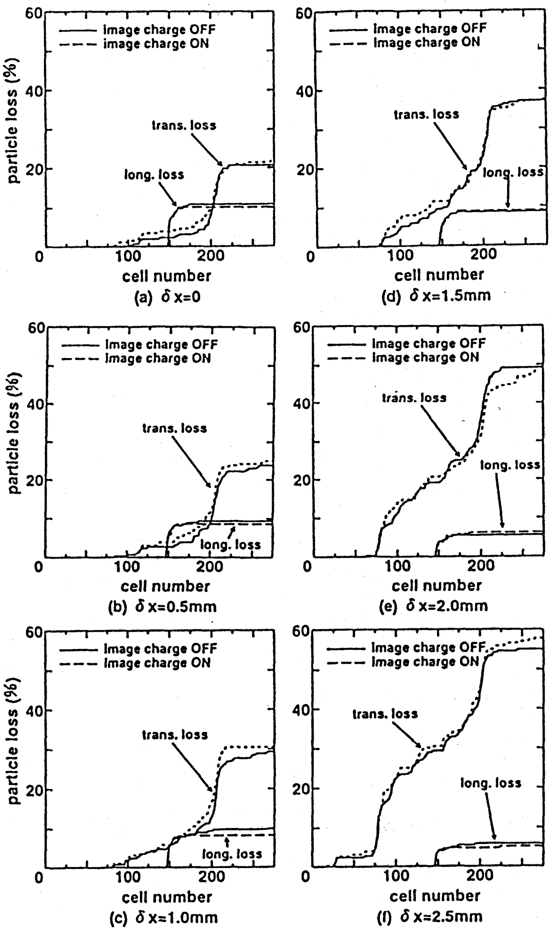

Fig.1 Beam particle loss profile

image charge force into account, the transverse loss of beam particles was principally increased, although there was no obvious increase of the longitudinal loss.

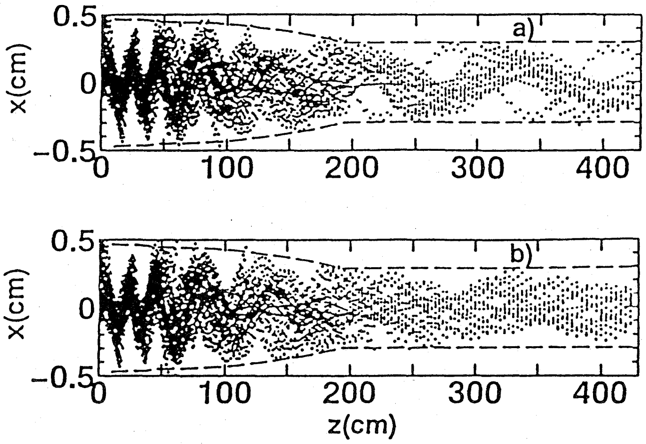

Fig.2 Simulation of beam trajectory
(δx=2.0mm)
a) image charge force ON
b) image charge force OFF

Fig.2 shows the simulation of particle trajectories when the beam is injected into the RFQ with a displacement of δx=2.0mm. a) indicates the result obtained when image charge force is included and b) when image charge force is neglected. Due to the effect of the image charge force, the coherent transverse beam oscillation is enhanced.

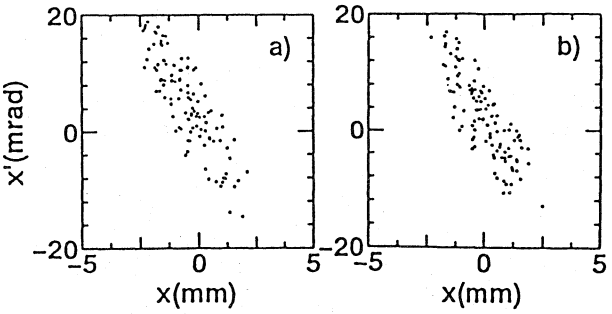

Fig.3 Output emittance (δx=2.0mm)
a) image charge force ON
b) image charge force OFF

Fig.3 shows the output beam emittance for the above mentioned injection condition (δx=2.0mm). Corresponding to the enhancement of the transverse oscillation caused by the image charge force, the particle distribution has a tendency to deviate from the center of the $x - x'$ phase plane, which can be seen from the output emittance.

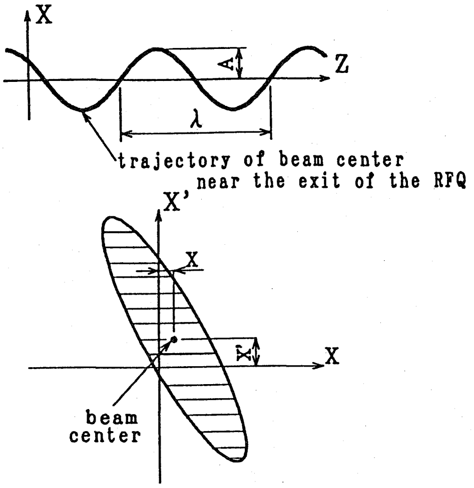

Fig.4 Schematic of the beam parameter used to derive the amplitude of oscillation

The aggravation of coherent beam oscillation caused by the image charge force was analyzed by estimating its amplitude. As shown in Fig.4, the oscillation near the exit of the RFQ can be expressed as

$$x = A\cos\kappa z, \tag{5}$$

$$\kappa = \frac{2\pi}{\lambda},$$

where A represents the amplitude, κ denotes the wave number, and λ is the wave length. The wave length λ, in fact, depends on z but was assumed to be constant near the exit of the RFQ in order to simplify the calculation. The angle of the beam trajectory is then given by:

$$x' = \frac{dx}{dz} = -A\kappa\sin\kappa z. \tag{6}$$

Following Eq.(5) and (6), the amplitude A can be estimated from

$$x^2 + \left(\frac{x'}{\kappa}\right)^2 = A^2. \tag{7}$$

Eq.(7) corresponds to an ellipse in the output emittance plane. Therefore, we can get x and x' from the average of the particle distribution. In this way the amplitude A can be solved out. Fig.5 shows the amplitude of coherent beam oscillation at the exit of the RFQ as a function of misalignment. Although these data are dispersive, the enhancement of the oscillation amplitude caused by the image charge forces can be clearly seen. The dynamics of beams in an RFQ is very complicated; to understand the factors affecting it, further investigations are necessary.

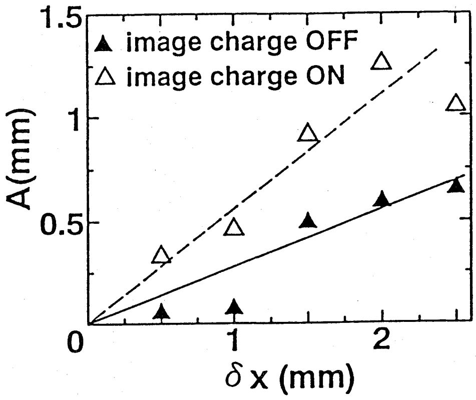

Fig.5 Amplitude of coherent beam oscillation versus misalignment

CONCLUSION

From the analysis of the behavior of the off-centered beams it was observed that

1. The image charge force reduces the transmission. However, the relative degradation of the beam transmission is almost the same in aligned and misaligned injection.

2. Due to the effects of the image charge force the amplitudes of betatron oscillation of off-centered beams in the RFQ are increased.

ACKNOWLEDGEMENTS

The authors thank Dr. Masahiro Okamura for useful discussions and Dr. Alex Zounek for critical reading of the manuscript.

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