Comparison of undulator-based and crystal-based positron sources

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1. Conventional positron sources are based on a cascade shower process initiated by an electron with energy E_0 in the amorphous converter.

The number of positrons in the maximum of the shower curve:

$$N_{e^+} \sim \frac{1}{3}\left(N_{e^+} + N_{e^-} + N_{ph}\right) = \frac{1}{3} N_{tot} = \frac{1}{3}\,\frac{E_0}{2E_{cr}}$$

$$E_{cr} = \frac{610 \text{ MeV}}{Z + 1.24}$$

For $E_0 = 30$ GeV in an iron ($E_{cr} \approx 22$ MeV) in the shower maximum ($t \approx 6X_0$)

$N_{e^+} \sim 90$ (EGS4), $N_{e^+} \sim 220$ (rough estimation)

Positron source at SLC — $E_0 = 30$ GeV, $t = 6X_0$

Energy efficiency $f = \dfrac{N_{e^+} \text{ accept.}}{N_{e^-} \cdot E_{e^-}}$

(see L. Rinolfi, T. Kamitani LC 2002)

SLC $f = 0.05$; CLIC (project) $f = 0.30$

It means the „energy cost" for each positron

$E_c = 20$ GeV (SLC), $E_c = 3.3$ GeV (CLIC)

The main problem is connected with huge energy deposition in a thick converter

2. TESLA e^+ source

Positrons are generated by undulator radiation in a thin rotatable convertor ($t = 0.4 X_0$)

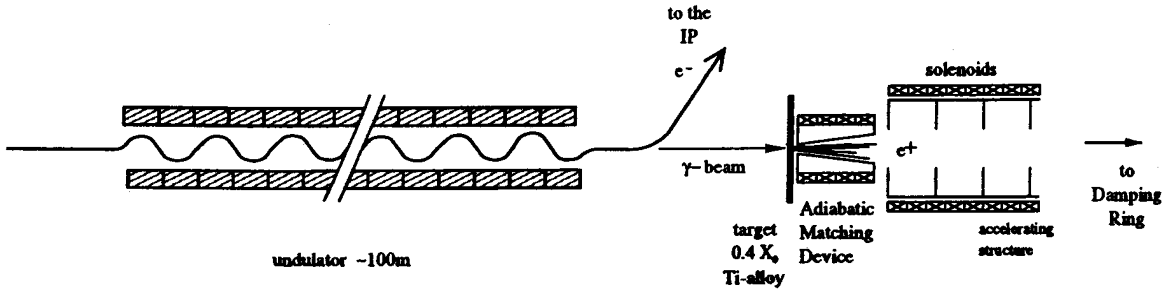


Figure 4.3.1: *Sketch of the positron source layout.*

parameter	SLC	TESLA
No. of positrons per pulse	(3–5) $\times 10^{10}$	5.6×10^{13}
No. of bunches per pulse	1	2820
pulse duration	3 ps	0.95 ms
bunch spacing	8.3 ms	337 ns
repetition frequency	120 Hz	5 Hz

Table 4.3.1: *Comparison of TESLA and SLC positron source parameters.*

gap [mm]	λ_u [cm]	B_{max} [T]	E_1 [MeV]
4.0	1.25	0.85	31.6
4.5	1.34	0.8	29.5
5.0	1.42	0.75	27.8

Table 4.3.2: *Parameters of the planar undulator.*

$$K = 0.934 \cdot B[T] \cdot \lambda_u [cm] = 1$$

$$L_u = 100\,m \qquad N_u = \frac{L_u}{\lambda_u} \approx 7000$$

Undulator	
peak field	0.75 T
period length	14.2 mm
gap height	5 mm
γ-spot size on target	0.7 mm
photon beam power	135 kW
Target	
material	Ti-alloy
thickness	1.42 cm ($0.4X_0$)
pulse temperature rise	420 K
av. power deposition	5 kW
Adiabatic Matching Device	
initial field	6 T
taper parameter	30 m^{-1}
end field	0.16 T
capture cavity iris radius	23 mm
General	
capture efficiency	16%
No. of positrons per electron	2
norm. e^+-beam emittance	0.01 m
total energy width	±30 MeV
required D.R. acceptance	0.048 m

Table 4.3.3: *Overview of the positron source main parameters.*

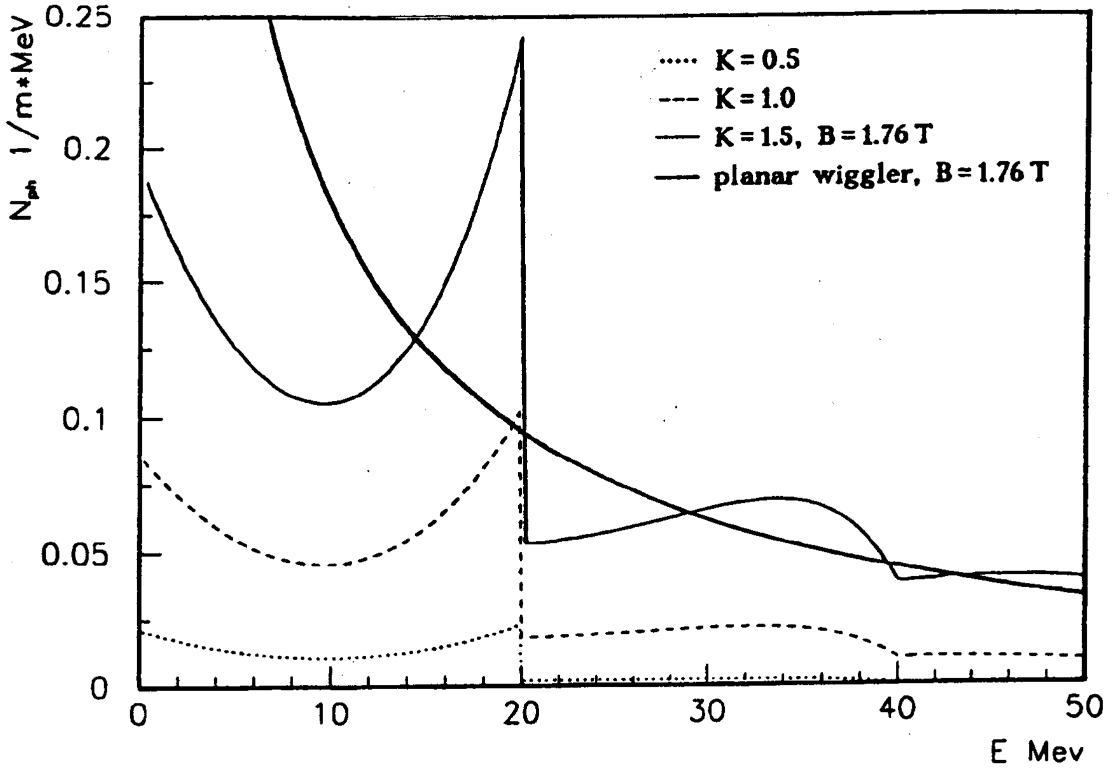


Fig. 1.5 Photon number spectrum for helical undulators and a planar wiggler, E=250 GeV. We will discuss the spectrum of a helical undulator in more detail in ch. 2.6.0.

Radiation losses $\Delta E_{rad} = 3$ GeV

The proposed scheme may provide 2 accepted positrons per initial electron.

The „energy cost" is 1.5 GeV only

due to large number of the emitted „soft" photons.

$$N_{ph} \sim 2\pi\alpha N_u \frac{K^2}{1+K^2} \approx 160 \; ph/e^-$$

The exact calculations (K. Flottmann, DESY-93-161)

$$N_{ph} = [3.56 - 0.69K]\, K^2 \cdot N = 200 \; ph/e^-$$

The average photon energy

$$\langle E_{ph} \rangle = \frac{\Delta E_{rad}}{N_{ph}} = 15 \text{ MeV}$$

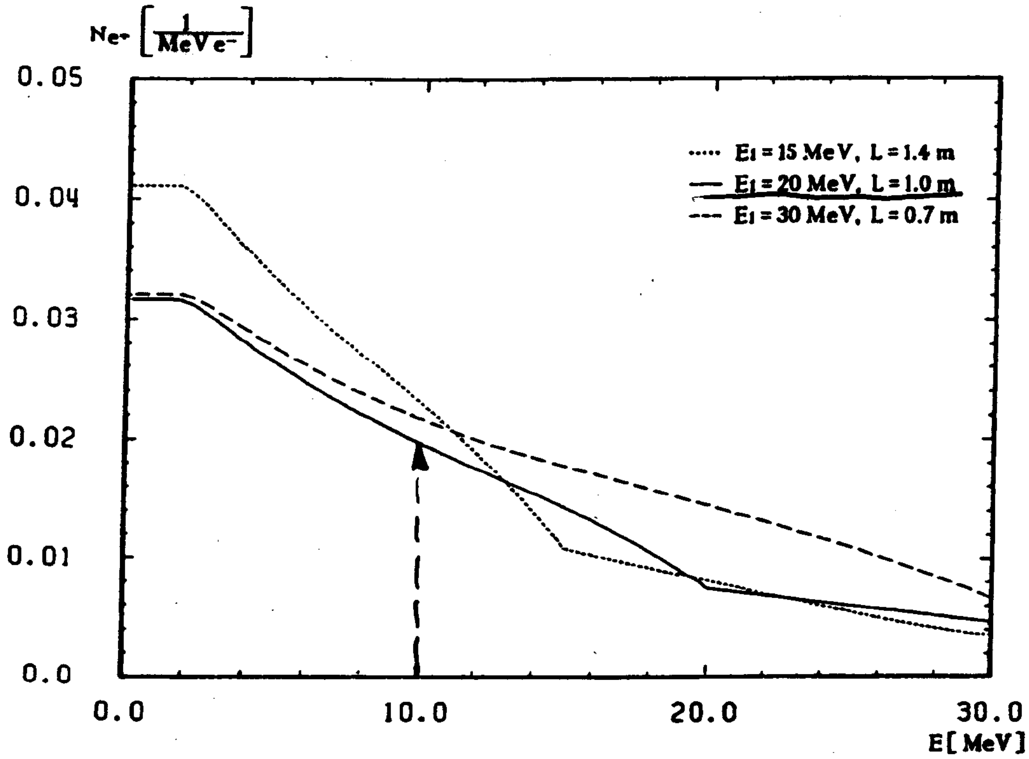


Fig. 1.9 Positron spectra for various E_1; K = 1, E = 250 GeV

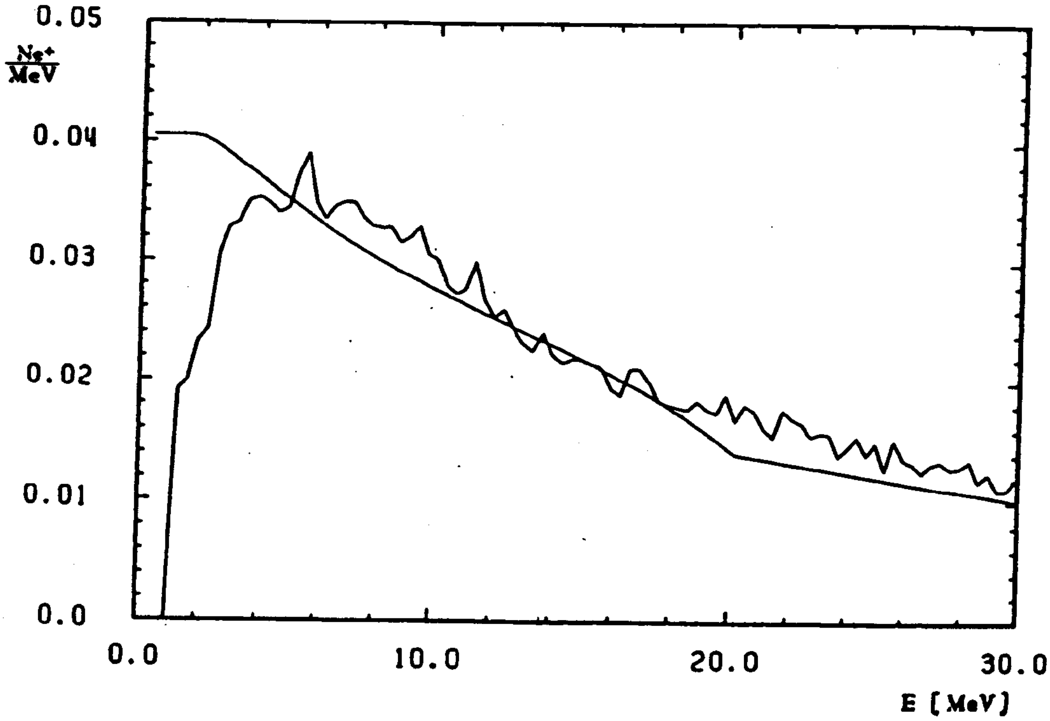


Fig. 1.10 Comparison of positron spectra calculated by means of the approximative method and by means of the shower code EGS 4

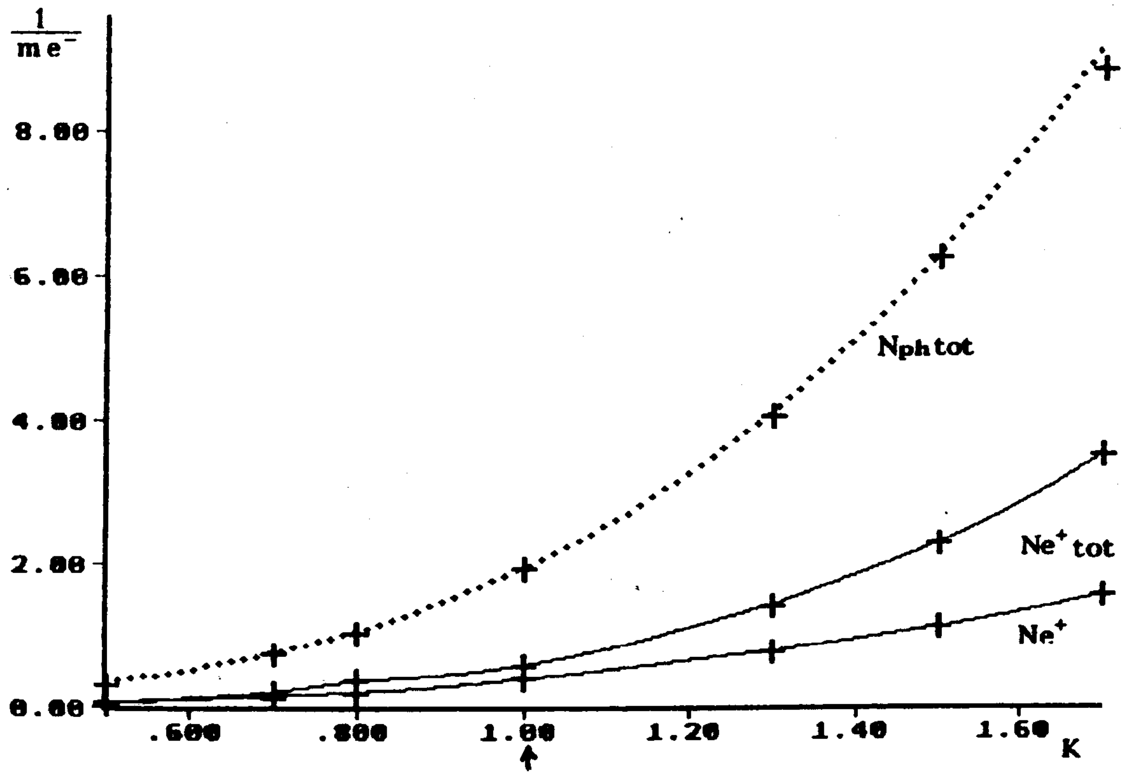


Fig 1.7 Dependence of the photon production and the positron production per radiation length on the parameter K, $E_1 = 20$ MeV, $E = 250$ GeV; dotted line: approximation according to eq. 1.9

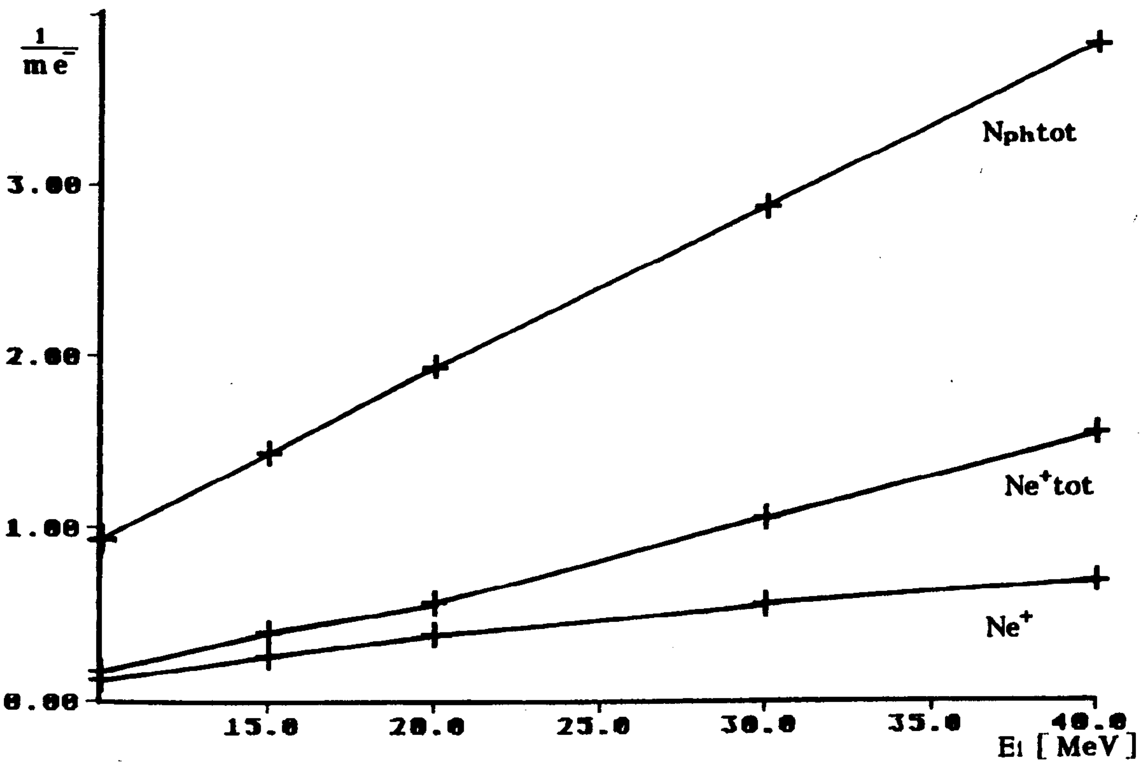


Fig. 1.8 Dependence of the positron production per radiation length on E1; $K = 1$, $E = 250$ GeV

3. A crystal may be considered as some kind of a solid-state undulator.

Is it possible to use one as a photon emitter?

In principle, yes.

Low energy experiments (Tomsk, Kharkov, INS) demonstrated the possibility to use crystal targets with thickness $t \gg L_{dechanneling}$ to produce a radiation enriched by „soft" photons.

For usual bremsstrahlung

$$\langle \omega_{BS} \rangle = \frac{\Delta E_{rad}}{\langle N_{ph} \rangle}$$

$$\Delta E_{rad} \approx \frac{t}{X_0} \int_{\gamma\omega_p}^{E_0} d\omega = \frac{t}{X_0} E_0 , \quad t \lesssim X_0$$

ω_p is plasmon energy ($\omega_p = 30\,eV$ for Si)

$$\langle N_{ph} \rangle = \frac{t}{X_0} \int_{\gamma\omega_p}^{E_0} \frac{1}{\omega} d\omega = \frac{t}{X_0} \ln \frac{mc^2}{\omega_p} \sim 10 \frac{t}{X_0}$$

$$\langle \omega_{BS} \rangle \approx 0.1\, E_0$$

For channeling radiation

$$u = \frac{\omega_{ch}}{E_0 - \omega_{ch}} \approx \frac{2\gamma\theta_c \frac{\lambda\!\!\!^{-}_e}{a_s}}{1 + \rho/2} = \frac{2\gamma^{1/2}\sqrt{\frac{V_0}{mc^2}}\,\frac{\lambda\!\!\!^{-}_e}{a_s}}{1 + \frac{\gamma V_0}{2mc^2}}$$

(see V. Baier et al. NIM B 103 (1995) 147)

For Si, $E_0 \sim 1\,GeV$, ⟨111⟩

$\omega_{ch} \sim 0.015\, E_0 \sim 15\, MeV$ ≪ …

For coherent bremsstrahlung

$$u = \frac{\omega_{CBS}}{E_0 - \omega_{CBS}} \sim \frac{\pi \gamma \theta}{a}$$

If $\langle \theta_{ms} \rangle > \theta_c$ then one may consider $\langle \theta_{ms} \rangle$ as an orientation angle

For 10 mm Si , $E_0 = 1$ GeV

$$\omega_{CBS} \sim 0.05 E_0 \sim 50 \text{ MeV}$$

4. For axial orientation radiation losses consist of two parts :

$$\Delta E_{ax} = \Delta E_{BS} + \Delta E_{cr}$$

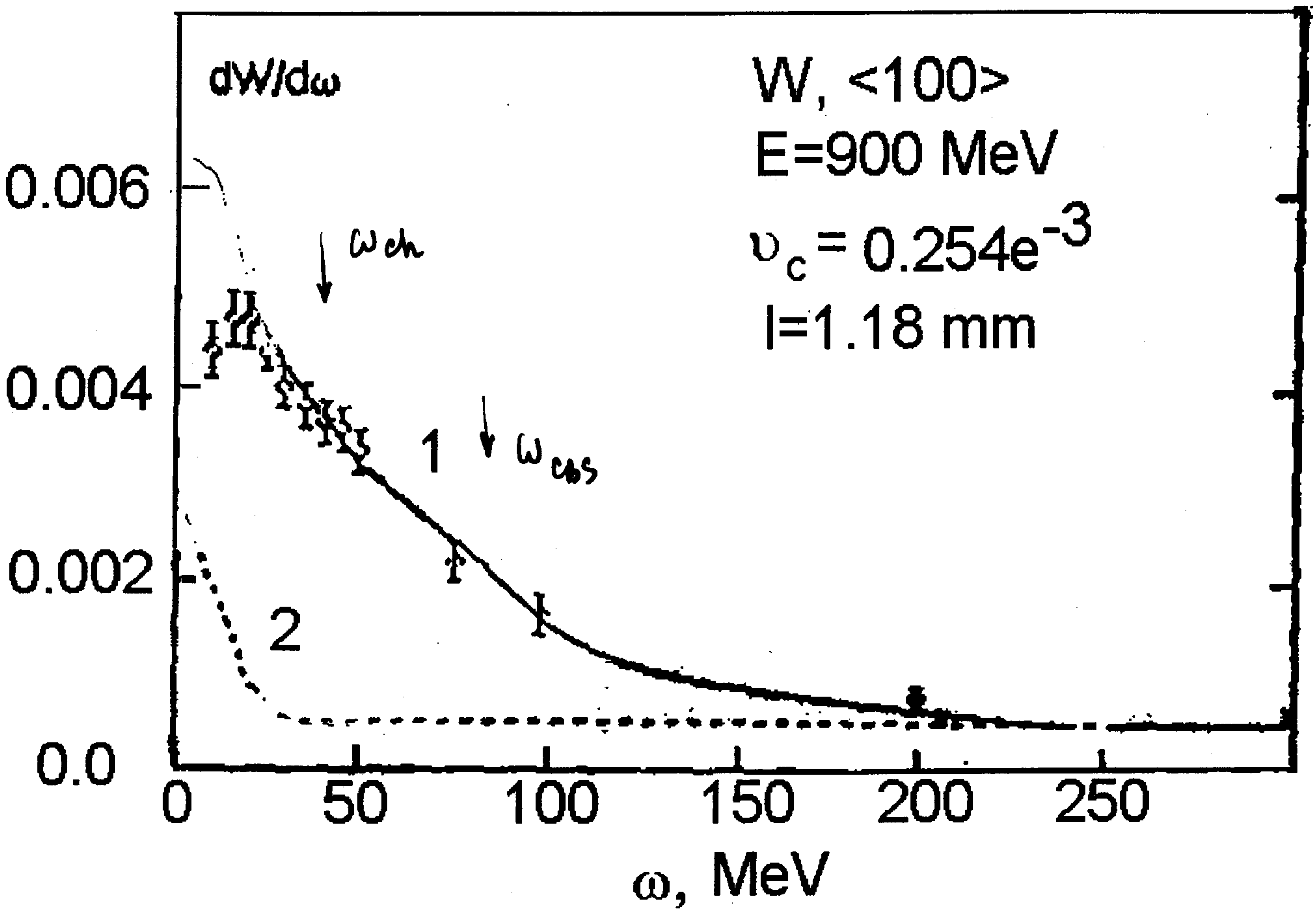
For $E_0 = 0.9$ GeV (Tomsk's data)

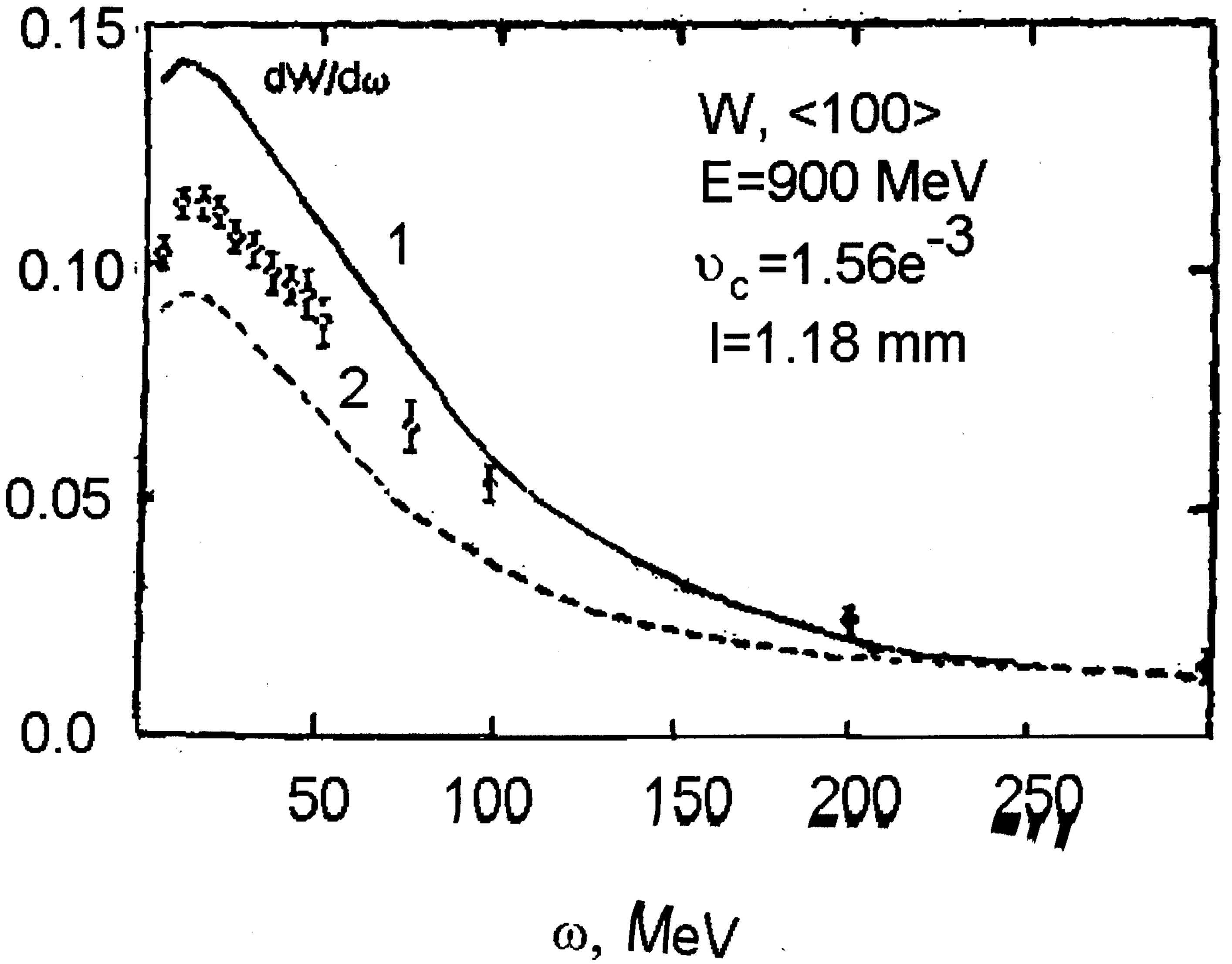
	diamond 10 mm	Si 10 mm	Tungsten 1.2 mm
$\frac{\Delta E_{cr}}{\Delta E_{BS}}$	~ 2.5	~ 1.8	~ 1.5

$$\langle N_{ph}^{cr} \rangle \approx \frac{\Delta E_{cr}}{\langle \omega_{cr} \rangle}$$

Even for $E_0 \sim 1$ GeV

$$\langle N_{ph}^{cr} \rangle > \langle N_{ph}^{BS} \rangle$$


dW/dω
W, <100>
E=900 MeV
υ_c = 0.254e^{-3}
l=1.18 mm
0.006
0.004
0.002
0.0
ωch
ωcbs
1
2
0
50
100
150
200
250
ω, MeV


0.15
dW/dω
W, <100>
E=900 MeV
υ_c =1.56e^{-3}
l=1.18 mm
1
2
0.10
0.05
0.0
50
100
150
200
250
ω, MeV

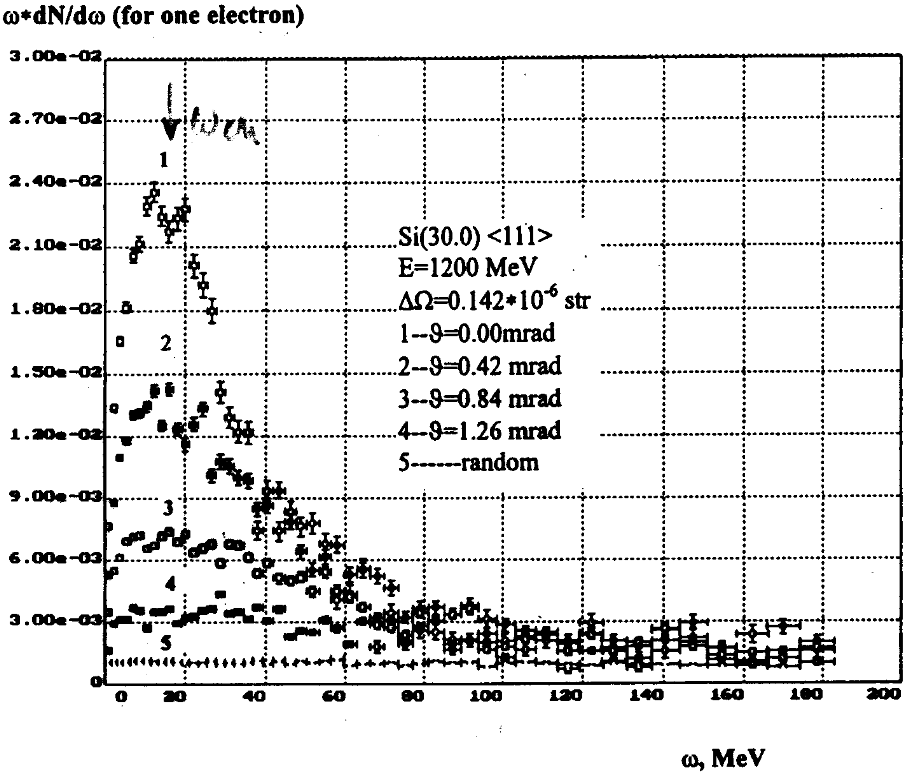


Fig. 3. Spectral-angular distribution of gamma-radiation for the silicon single crystals ⟨1 1 1⟩ with thickness 30 mm for electrons with energy $E = 1200$ MeV in solid angle $\Delta\Omega = 0.142 \times 10^{-6}$ sr.

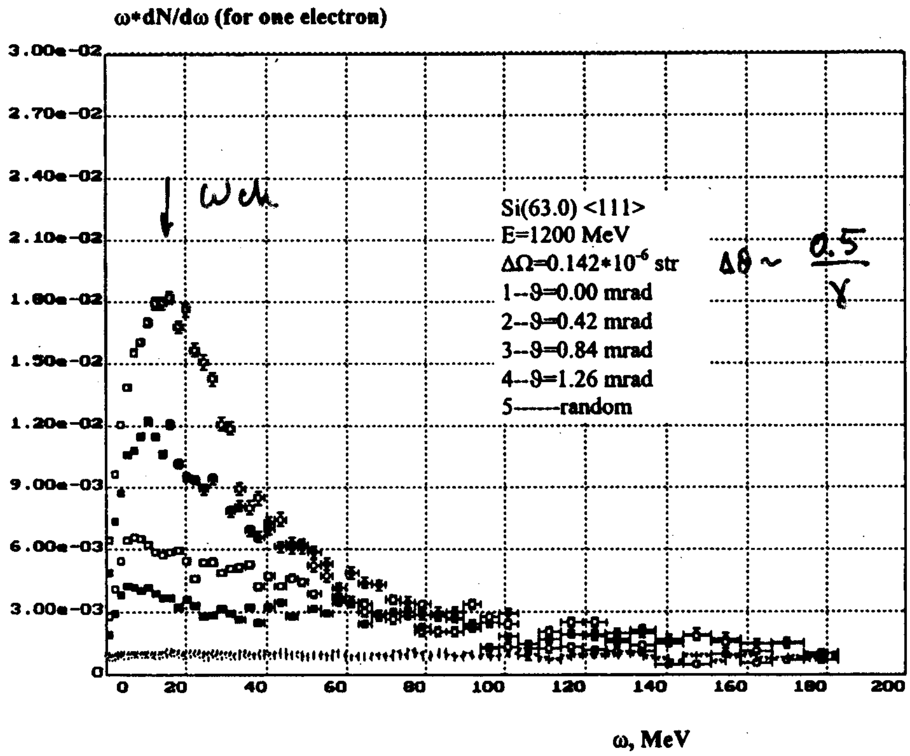
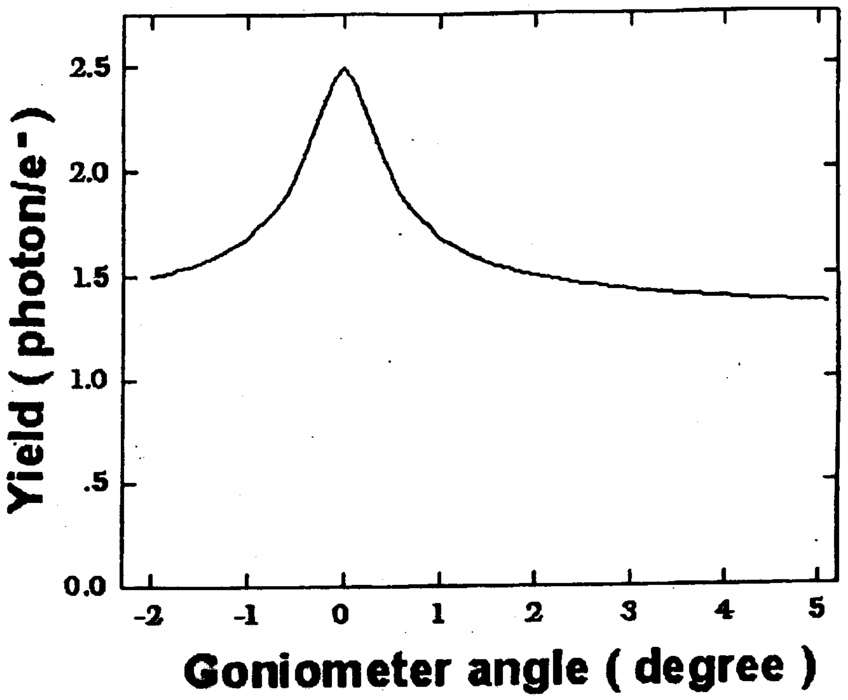


Fig. 4. Spectral-angular distribution of gamma-radiation for the silicon single crystals ⟨1 1 1⟩ with thickness 63 mm for electrons with energy $E = 1200$ MeV in solid angle $\Delta\Omega = 0.142 \times 10^{-6}$ sr.



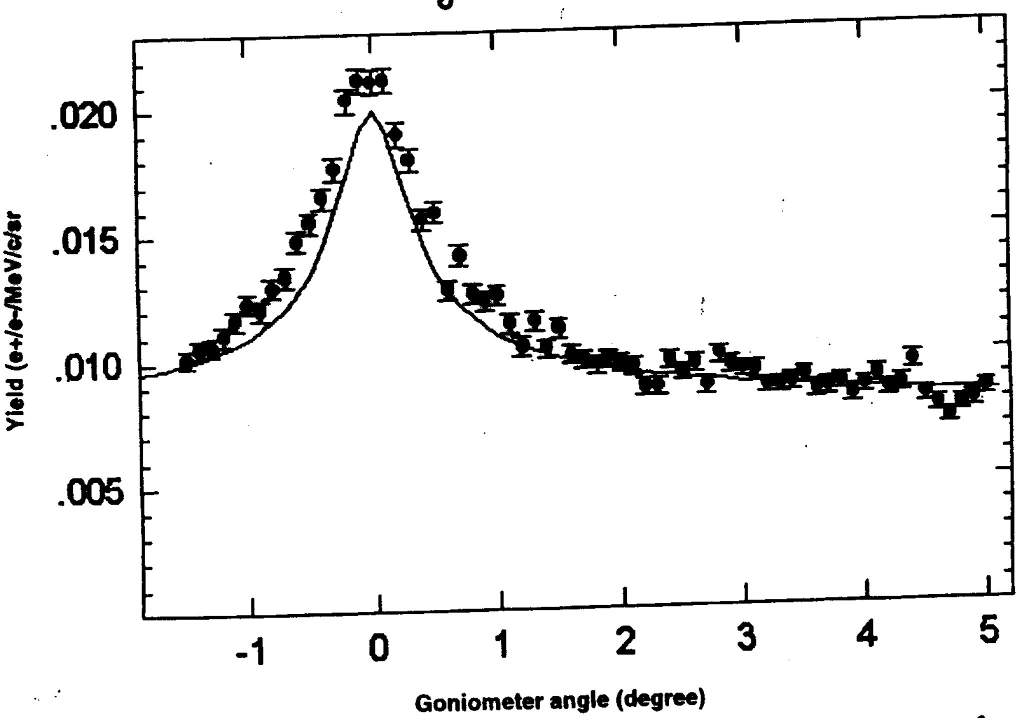
$E_e = 1.2$ GeV

$t = 1.2$ mm W

CBS only

Fig. 9. Calculated orientation dependence of photon yield with the energy greater than 21 MeV.

Positron yield ($p = 20$ MeV/c)



Contribution from coherent bremsstrahlung only

E = 1.0 GeV

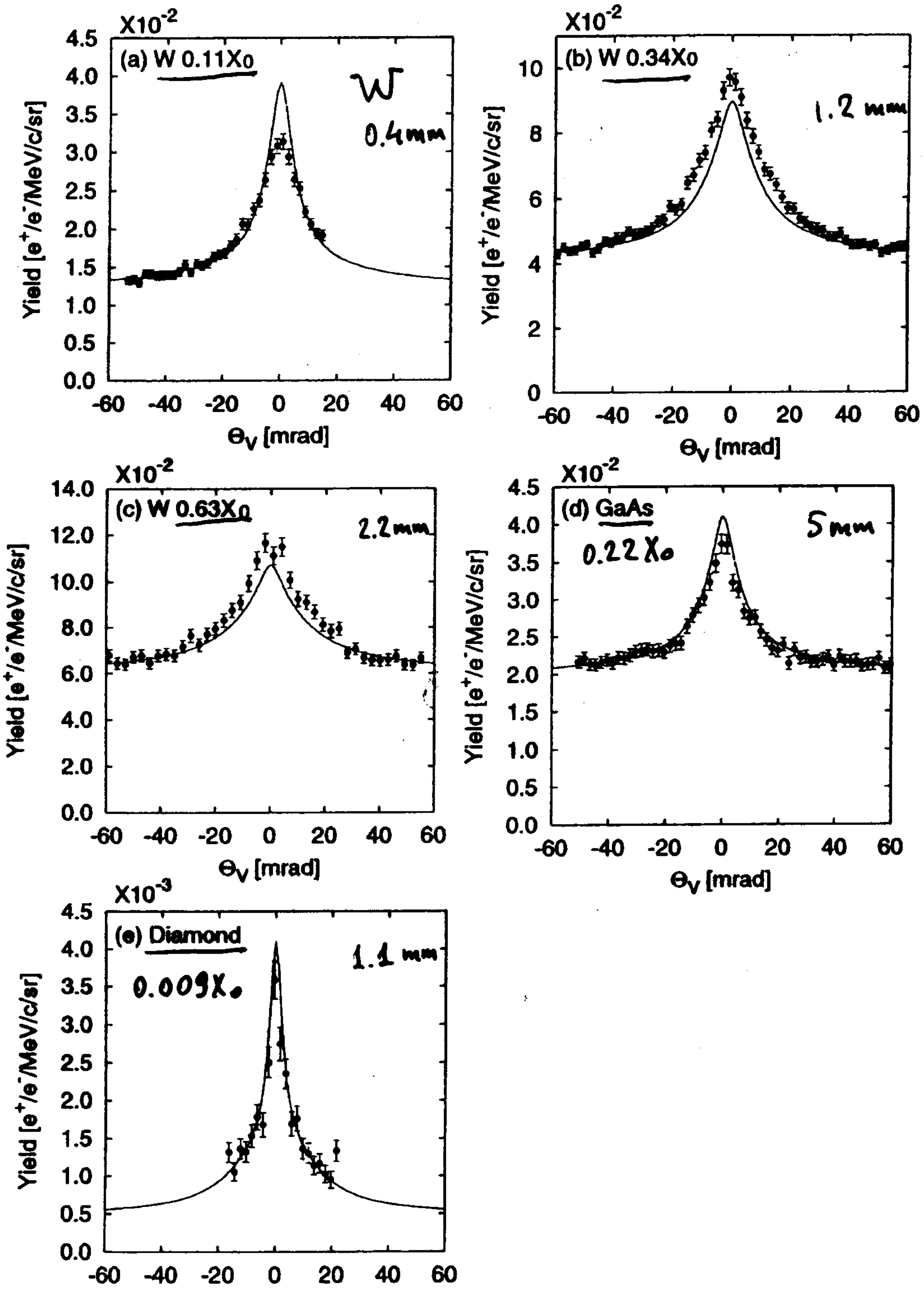
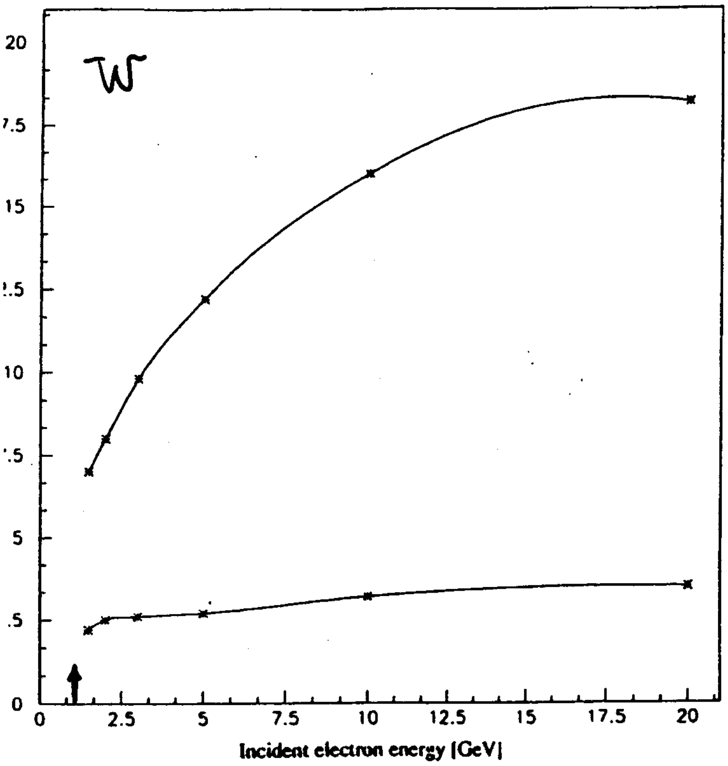


Fig. 12. Rocking curves of positron yields at $E_{e^-} = 1.0$ GeV, $\theta_{e^+} = 0°$ and $P_{e^+} = 20$ MeV/c. Target: (a) W with t = 0.4 mm (0.11 X_0), (b) W with $t = 1.2$ mm (0.34 X_0), (c) W with $t = 2.2$ mm (0.63 X_0), (d) GaAs with $t = 5.0$ mm (0.22 X_0) and (e) diamond with $t = 1.1$ mm (0.009 X_0). The curves shown in the figures are the results of simulations based on the coherent bremsstrahlung process.



. 2. Photon yield versus incident electron energy for 1 mm :kness; cut-off energy = 1 MeV. Upper figure: crystal, lower figure: amorphous.

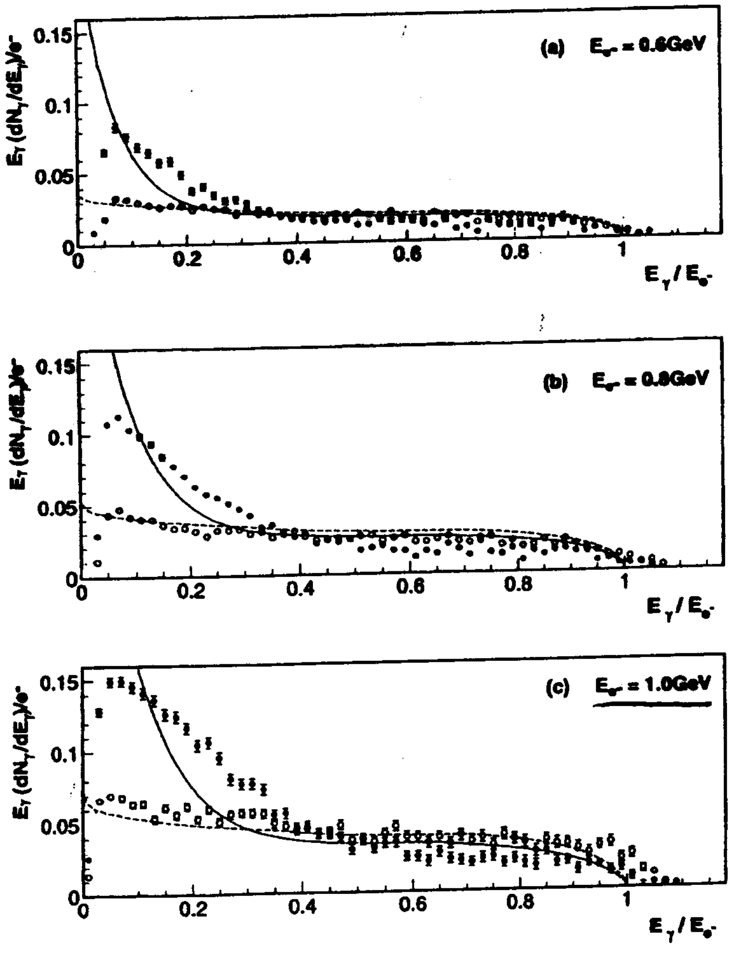
Simulation (X. Artru et al.)

$\frac{I_0}{I_{rand}} \approx 3$

$\langle N_{ph} \rangle \sim 10$ for $E = 4$ GeV

1.2 mm

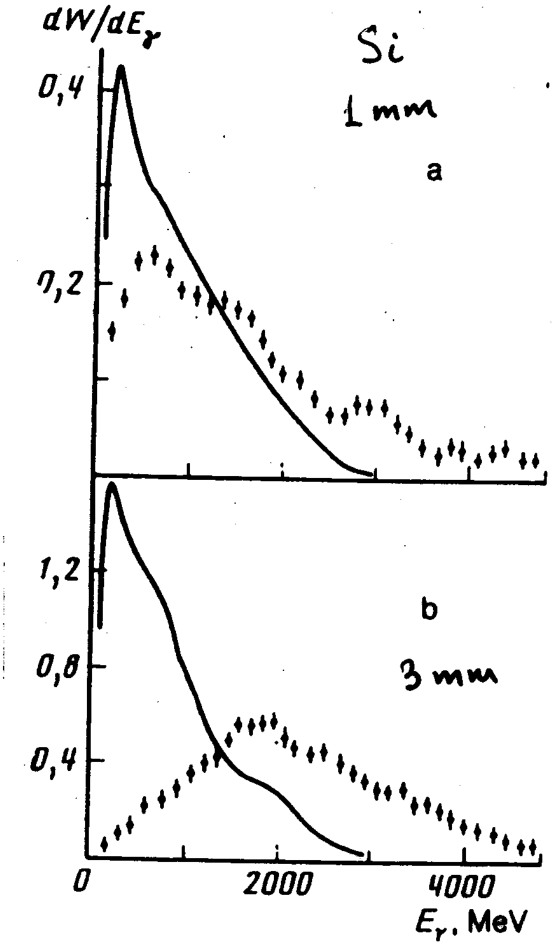
W

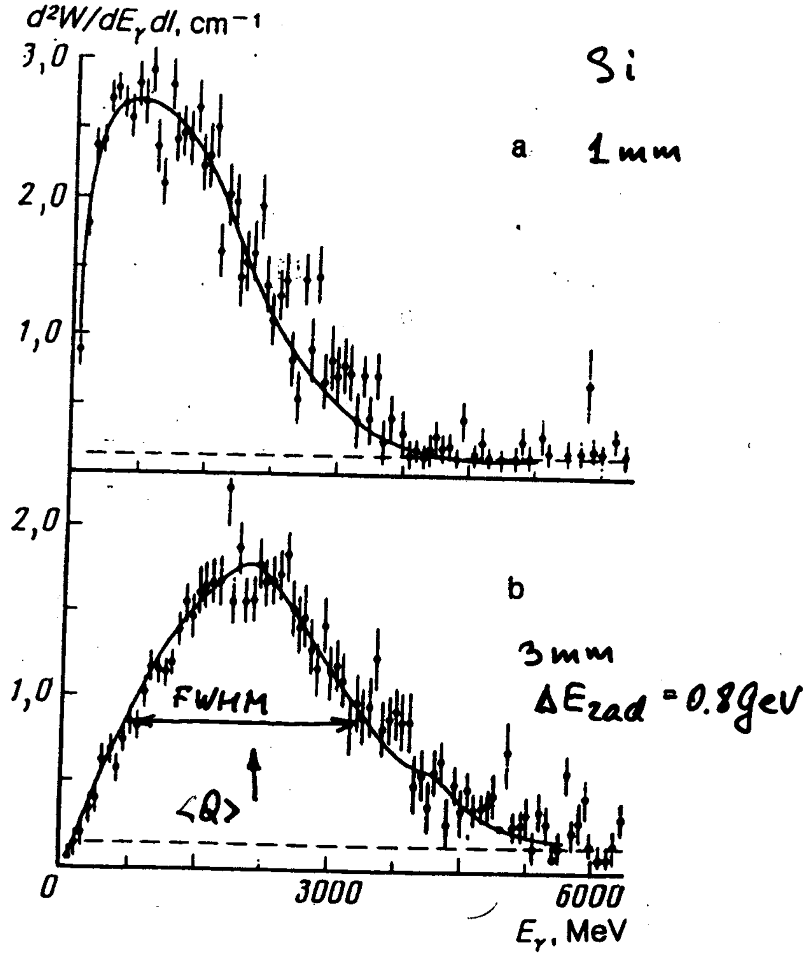


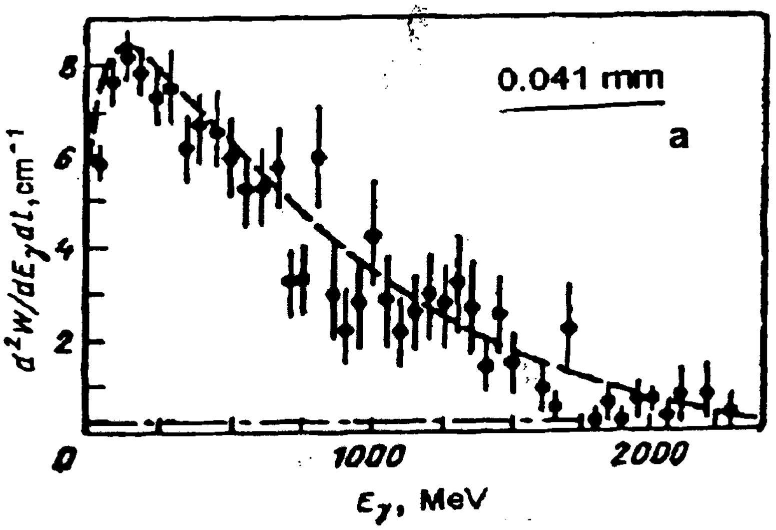
Experiment

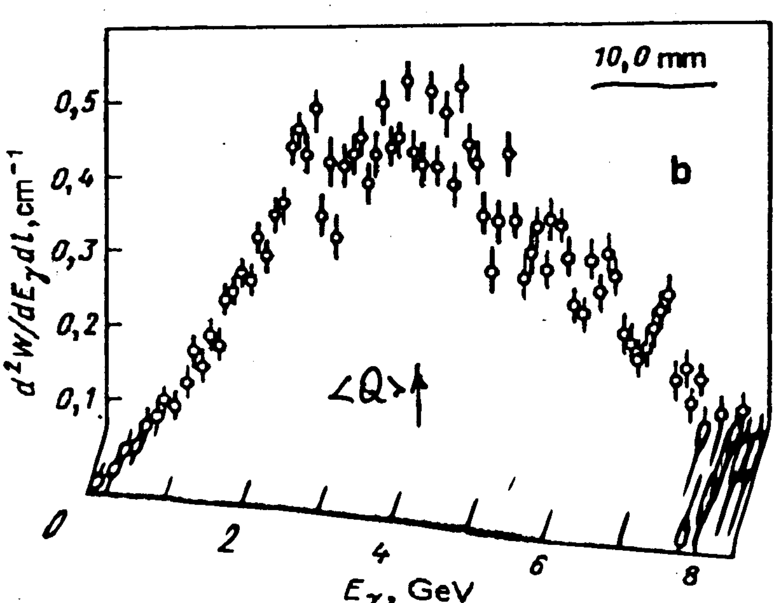
$\frac{I_0}{I_{random}} \approx 2.3 \pm 0.14$

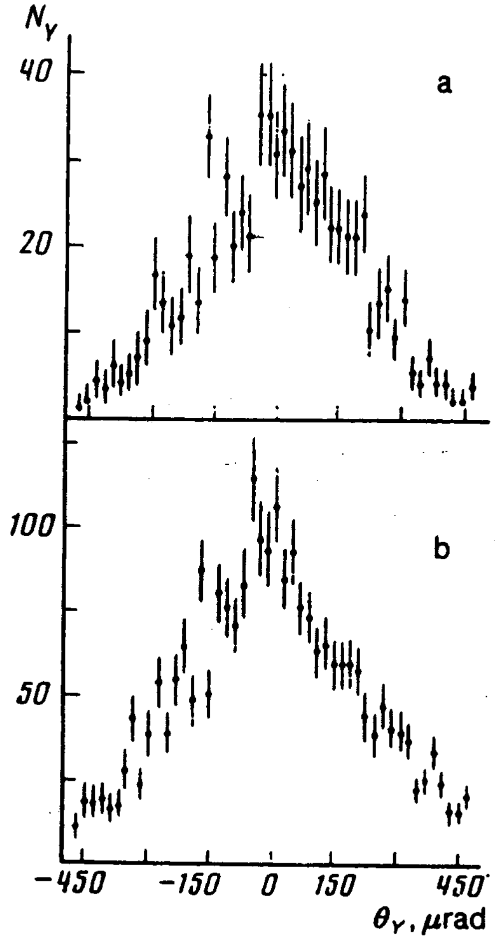
E = 10 GeV (A. Vodopianov et al., JINR, Russia)











Channeling electrons

$\theta_e \leq \theta_c$

$\theta_e \leq 3\theta_c$

FIG. 4. Distribution of the exit angles of gamma photons from a silicon crystal 3.0 mm thick for events characterized by the following entry angles relative to the axis: a) from 0 to 130 μrad; b) from 0 to 400 μrad.

No significant difference

Estimation of a photon multiplicity from the energy loss distribution (energy straggling) – see A. Kolchuzhkin, A. Potylitsyn, NIMB 173 (2001) 126

If $\langle Q \rangle$ is the mean energy losses, σ is a variance of the energy distribution, then

$$\langle N_{ph} \rangle \sim \frac{\langle Q \rangle^2}{\sigma^2}$$

For Dubna's experiment, $E_0 = 10$ GeV

$t = 3$ mm, $\langle Q \rangle \approx 2.3$ GeV, $\sigma \approx \frac{FWHM}{2.36} = 1.1$ GeV

$\langle N_{ph} \rangle \approx 4$ ph/e-

$$\langle \omega \rangle \approx \frac{\Delta E_{rad}}{\langle N_{ph} \rangle} \approx \frac{0.8 \text{ GeV}}{4} = 200 \text{ MeV}$$

MC simulations (M.D. Bavizhev et al. Sov. Phys. JETP, v.68, (1989) 803)

TABLE I.

Crystal thickness, mm	Multiplicity of emission ($\omega_{thr} = 20$ MeV)		
	Experiments		Monte Carlo calculations
	$\theta_{in} = 0$–$800\ \mu rad$	$\theta_{in} = 0$–$130\ \mu rad$	
0,8	1,2	1,7	1,8
3,0	3,9	5,2	5,4

$t = 3$ mm

$t = 10$ mm, $\langle Q \rangle \approx 4$ GeV, $\sigma \approx 1.8$ GeV

$\langle N_{ph} \rangle \approx 5$ ph/e-

$$\langle \omega \rangle \approx \frac{1.2 \text{ GeV}}{5} \approx 240 \text{ MeV}$$

V. Baier, V. Katkov, V. Strakhovenko

Sov. Phys. Usp. 32 (1989) 972

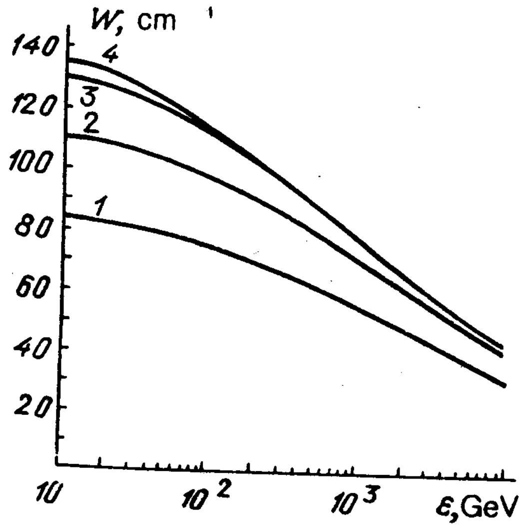


FIG. 2. Energy dependence of the total probability of radiation in Si ⟨110⟩ ($T = 293$ K, curve 1), in diamond ⟨111⟩ ($T = 293$ K, curve 2), in Ge ⟨110⟩ ($T = 280$ K, curve 3), and in Ge ⟨110⟩ ($T = 100$ K, curve 4).

For 3 mm Si the total photon number

$N_{tot} \approx 24$ ph / e^-

Many photons with $\omega < 1.02$ MeV

E = 4.5 GeV R. Avakian et al.

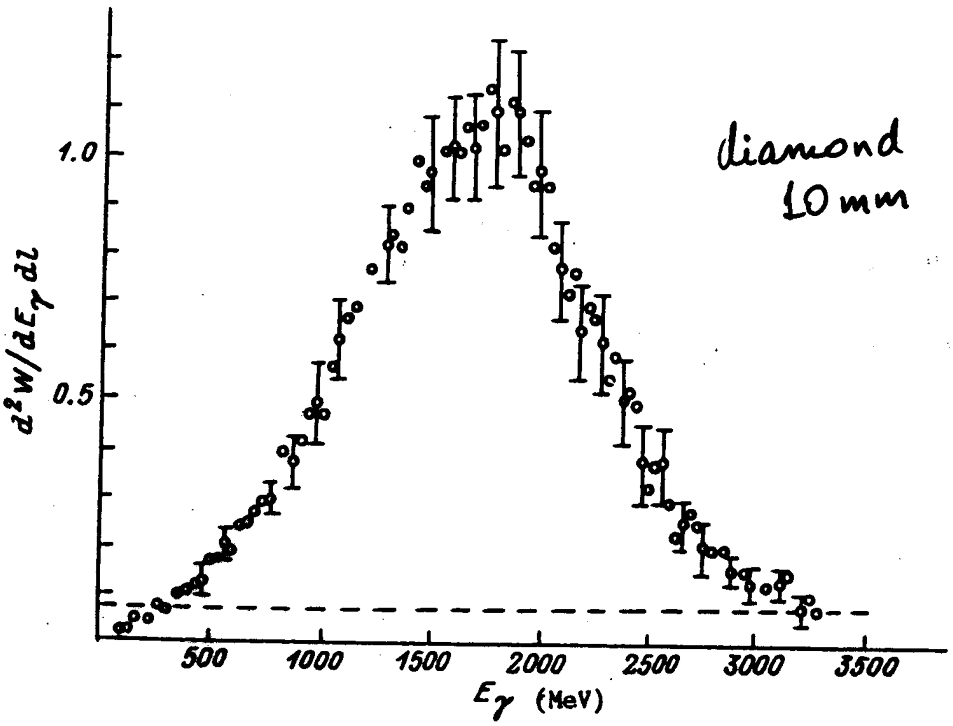


FIG. 1. Energy spectrum radiated by electrons during axial channeling.

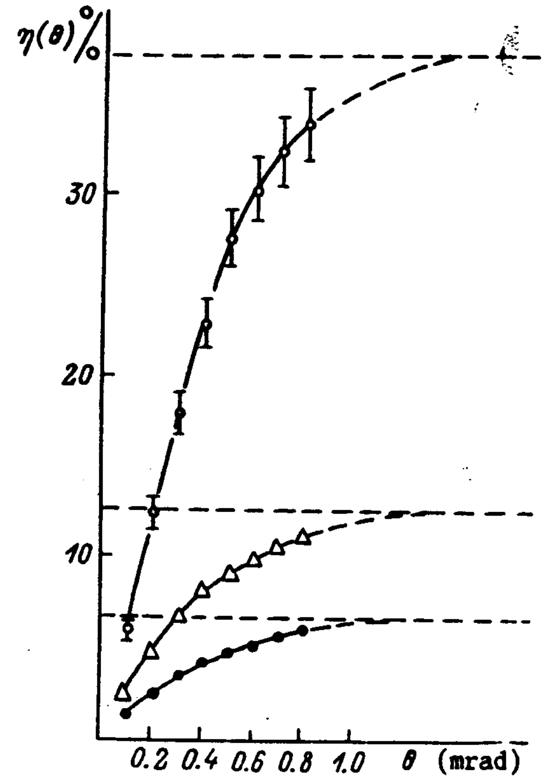
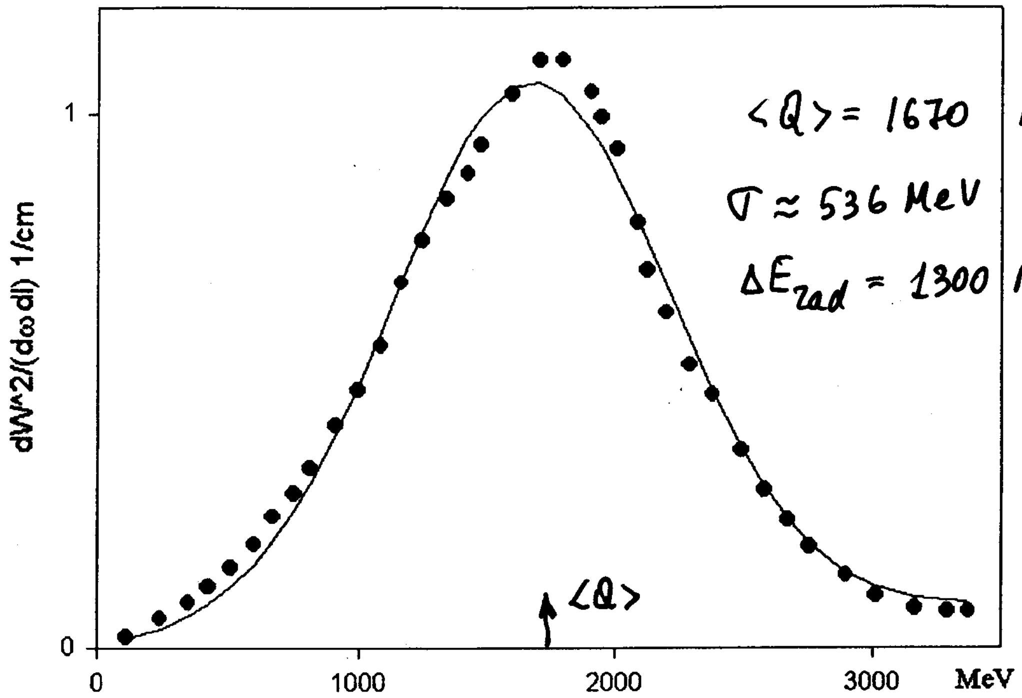


FIG. 3. Integrated radiation yield as a function of the photon emission angle. The horizontal dashed line shows the level of the radiation yield over all photon emission angles.



$\langle Q \rangle = 1670$ MeV

$\sigma \approx 536$ MeV

$\Delta E_{rad} = 1300$ MeV

$$\langle N \rangle \sim \frac{\langle Q \rangle^2}{\sigma^2} \sim$$

~ 10 ph/e^-

$\langle \omega_{ph} \rangle \sim 130$ MeV $\quad \omega_{ch} \sim 100$ MeV

Fig.3 ***Radiation energy losses distribution for 4.5 GeV electrons channeled along <111> axis in 10mm diamond crystal.***

- Diamond is the best crystal for generation of intense CBS beams.
- Thermal conductivity of diamond is very high
  660 W/m/K against 170 W/m/K (for tungsten)

- SLAC test for diamond target
  ($\tau_B \sim 1$ psec, $S_B \sim 0.06\ \mu m^2$, $N_e \sim 2\cdot 10^{10}$ e^-/bunch)
  No visible damage

4. Last experimental results (R. Hamatsu, QABP report) allow to think that scheme with light Z crystal photon emitter is more effective for electron energy $E_0 < 10$ GeV.

Let's estimate positron yield for combined target

diamond <111>, 15 ÷ 20 mm thickness
+ amorphous converter $t_{am} = 1 X_0$

and electrons with $E_0 = 5$ GeV.

For this diamond thickness one may expect

$$\Delta E_{rad} \approx 0.5 E_0 = 2.5 \text{ GeV}$$

$$\langle \omega \rangle \sim 150 \text{ MeV}, \quad \langle N_{ph} \rangle \sim 17 \text{ ph}/e^-$$

After passing through crystal the initial electron will have energy $E_- = E_0 - \Delta E_{rad} = 2.5$ GeV

In amorphous converter e^+ will be produced by this electron and a number of „soft" photons.

Positron spectrum for first case and thin target ($t_{am} \lesssim X_0$) may be calculated analytically (see A. Potylitsyn, NIMA, v. 398 (1999) 395)

$$\frac{dN_+}{dE_+} \approx 0.07 \frac{t^2}{X_0^2} \left[\ln\left(\frac{1}{\lambda}\right) - 0.5 \right] \left(\frac{1}{E_+} - \frac{1}{E_-} \right), \quad \left[\frac{1}{\text{MeV}} \right]$$

$$\lambda = \frac{Z^{1/3}}{111}$$

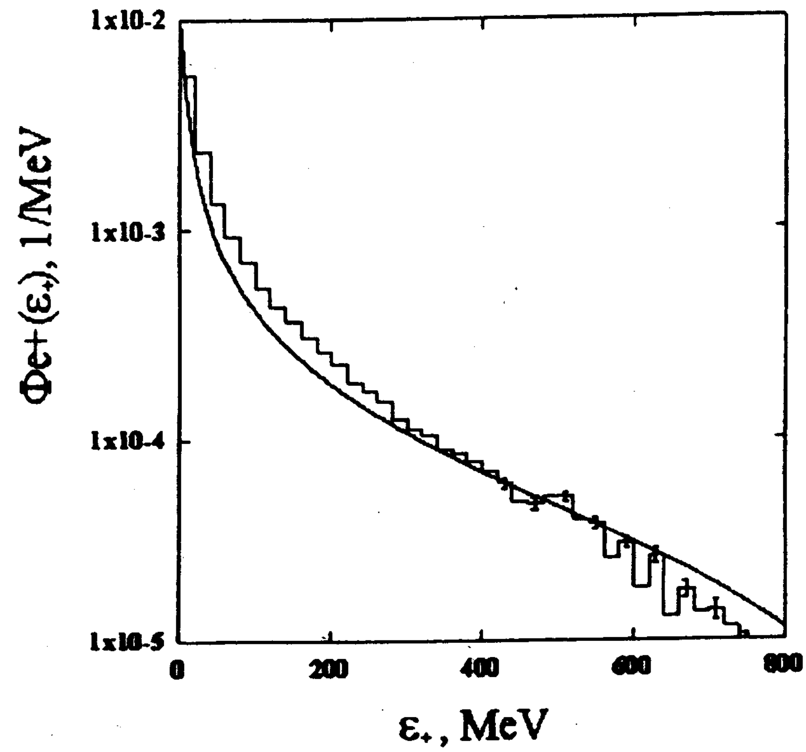


Fig. 2. Positron spectrum generated by electron with energy $E = 1$ GeV in amorphous target of the thickness $t = 0.5$ rad. length. (Histogram) Monte Carlo simulations [7], (curve) calculations from Eq. (5).

The similar approach may be developed for initial photons.

Let's consider model „flat" photon spectrum (closed to UR one):

$$\frac{dN_{ph}}{d\omega} = \begin{cases} \frac{N_{ph}}{\omega_{max}}, & \omega \le \omega_{max} \\ 0 & \omega > \omega_{max} \end{cases}$$

In this case

$$\Delta E_{rad} = \int_0^{\omega_{max}} \omega \frac{dN}{d\omega} d\omega = N_{ph} \frac{\omega_{max}}{2} = N_{ph} < \omega >$$

From here $\omega_{max} = 2 < \omega >$

After convolution with this spectrum it is possible to obtain analytycal expression for positron spectrum:

$$(*) \quad \frac{dN_+}{dE_+} \approx 0.14 \frac{t}{X_0} \frac{N_{ph}}{\omega_{max}} \ln\left(\frac{\omega_{max}}{E_+}\right)\left\{ \frac{1}{2} \ln\left[\frac{\omega_{max} E_+}{(mc^2)^2}\right] - 1.2\right\}$$

Flottmann's calculations for UR with

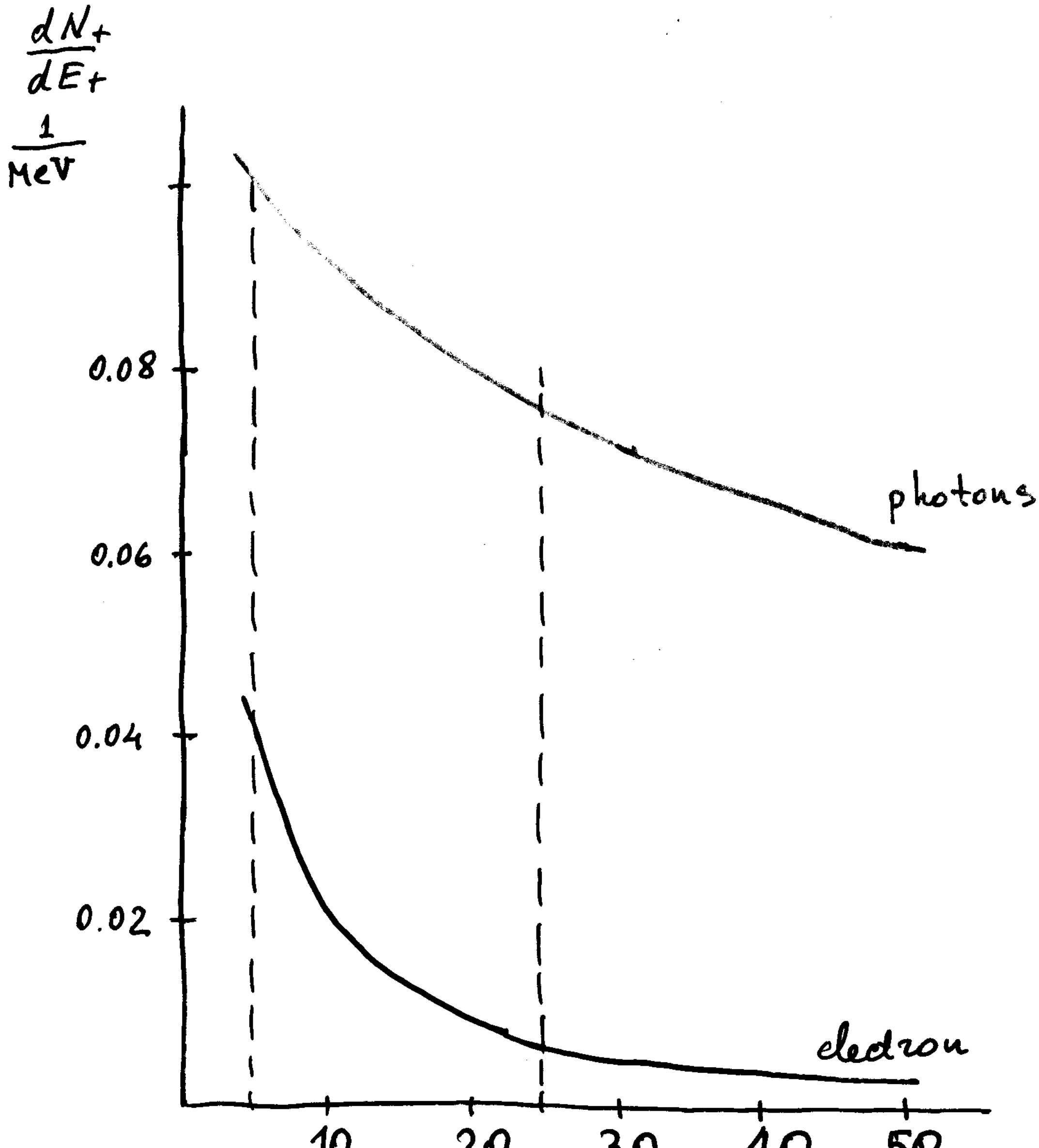
$\omega_{max} = E_1 = 21$ MeV, $N_{ph} = 2$ ph/e- ($L_u = 1$ m)

gives result $\frac{dN_+}{dE_+} = 0.18$ for $E_+ = 10$ MeV

From (*) one may obtain (for the same parameters):

$$\frac{dN}{dE_+} = 0.20$$

Positron spectra



Yield (5 MeV < E_+ < 25 MeV)

e^- : $\Delta N_+ = 0.32\ e^+/e^-$

photons: $\Delta N_+ = 1.74\ e^+/e^-$

$\Sigma \approx 2\ e^+/e^-$

Summary

- Scheme for positron generation with combined target looks more preferrable

- Thick diamond crystal is the best candidate for a photon emitter

- Problem of an artificial diamond producing with thickness 10÷20 mm may be resolved in the nearest future

- Measurements of radiation losses (energy distribution of electron beam passed through light Z crystal target) may be very important for comparison with theory

- Detailed 6-D simulation of crystal-based e^+ source is needful to choose configuration of source (E_0, Z, t_{cr}, t_{am}) for highest efficiency