Semiclassical Theory of Crystal-Assisted Pair Production: Beyond the Uniform Field Approximation

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Within the semiclassical theory of QED, a simple expression of pair production in strong field is obtained that approaches to the constant field approximation at the strong field limit while it gradually behaves like the Bethe-Heitler type when the field becomes weaker. By using the impact approximation for the calculation of trajectory, a simple expression is obtained. Though simple, it agrees well with the experimental result of crystal-assisted pair production.

1 Introduction

It is widely known that electron-positron pair production is well described by the Bethe-Heitler formula as far as the process can be regarded as the incident photon collides with an isolated atom. However, when photons enter a strong electromagnetic field, the perturbative approach such as the Bethe-Heitlers becomes inappropriate for describing the pair production process. For such strong field QED effect, Baier and Katokov[1] proposed a semiclassical expression for radiation

$$\frac{dN}{d\eta} = \frac{\alpha c}{\pi \lambda_0 \gamma}(J_c + J_s), \tag{1}$$

$$J_c = \int_0^\infty \left[1 + \frac{\gamma^2}{2}(\delta \boldsymbol{\beta}_\perp)^2\right] \sin\Delta \, \frac{d\tau}{\tau} - \frac{\pi}{2}, \tag{2}$$

$$J_s = \left(\frac{\gamma - \gamma'}{2\gamma\gamma'}\right) \int_0^\infty \frac{\gamma^2}{2}(\delta \boldsymbol{\beta}_\perp)^2 \, \sin\Delta \, \frac{d\tau}{\tau}, \tag{3}$$

where $\omega^* = (\gamma/\gamma')\omega$, $\eta = \hbar\omega/E$, $\delta\boldsymbol{\beta}_\perp = \boldsymbol{\beta}_\perp(t_+) - \boldsymbol{\beta}_\perp(t_-)$, $t_\pm = t_0 \pm \tau/2$, $E' = E - \hbar\omega$, $\gamma(\gamma') = E/(m_0c^2)(E'/(m_0c^2))$, E being the initial energy of the particle and m_0 its rest mass. The phase determined by *the trajectory* of a radiating particle is given by

$$\Delta(\tau) = \frac{\omega\tau}{2\gamma^2} - \frac{\omega}{2c^2\tau}(\delta\boldsymbol{\rho})^2 + \frac{\omega}{2}\int_{t_-}^{t_+} \boldsymbol{\beta}_\perp^2(\tau')d\tau', \tag{4}$$

where $\delta\boldsymbol{\rho} = \boldsymbol{\rho}(t_+) - \boldsymbol{\rho}(t_-)$, $\boldsymbol{\rho}(t)$ being the transverse coordinate. Though there are some discussion on the derivation of the Baier-Katokov formula [2, 3], this formula explains very well the radiation as well as pair production in strong fields.

2 "Th-trajectory" and pair production

The pair production probability is obtained by using the crossing symmetry for Eq.(5) [3].

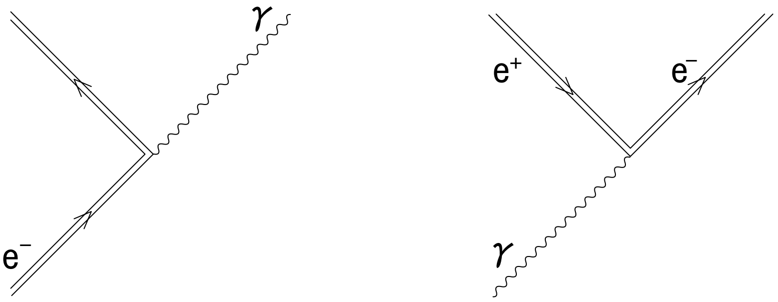

Figure 1: Crossing symmetry. The left-hand figure represents the channeling radiation while the right-hand one represents the pair production.

By changing the variables as

$$\begin{cases} E & \text{(initial energy)} & \rightarrow & -E_+ & \text{(produced positron)} \\ E' & \text{(final energy)} & \rightarrow & E_- & \text{(produced electron)} \\ \omega & \text{(emitted photon)} & \rightarrow & -\omega & \text{(absorbed photon)} \end{cases}$$

and multiplying the ratio of the density of final states,

$$\frac{E_+^2 dE_+}{(\hbar\omega)^2 d(\hbar\omega)},$$

we obtain the pair production probability for the th-trajectory approximation as well as the synchrotron approximation, though one may feel that the idea of particle trajectory is rather "spooky"[3]. We obtain,

$$\frac{dN_+}{d\eta_+} = \left(\frac{\alpha c}{\pi \lambda_0}\right)\left(\frac{m_0 c^2}{\hbar\omega}\right) J, \tag{5}$$

where $dN_+/d\eta_+$ represents the number of produced positrons, $\eta_+ = E_+/\hbar\omega$, E_+ being the energy of the produced positrons, α the fine-structure constant, and λ_0 the Compton wavelength.

The factor J for the CFA is given by

$$J_{\rm cf} = \frac{1}{\sqrt{3}}\left[\left(1 - \eta_+ + \frac{1}{1-\eta_+}\right) \mathrm{K}_{\frac{2}{3}}(\xi^*) - \int_{\xi^*}^{\infty} \mathrm{K}_{\frac{1}{3}}(\lambda) d\lambda\right], \tag{6}$$

where $\xi^* = 2/[3\eta_+(1-\eta_+)\chi]$ and $\chi = \hbar\omega\lambda_0 F(t_0)/(m_0 c^2)^2$ and $F(t_0)$ is the force by the field acting on the positron at time t_0. Eq.(6) is obtained by assuming that the trajectory of produced positrons are circular.

The CFA is applied to evaluate the pair production process when high-energy photons enters a crystal along the major crystal axis. It is reported that when photons are directed exactly parallel to the crystal axis, CFA agrees very well with the experimental result [5]. However, naturally, CFA does not explain the angular dependence of the pair production rate. For the purpose to calculate the angular dependence, the authors of Ref.[5] calculated the Baier-Katkov formula with some "numerical experiments" [6] and a good agreement of their numerical approach with the experimental result has been demonstrated. However, the process of the "numerical experiments" has not been described in their paper. Therefore, nobody can reproduce the calculations.

Recently, two of the present authors obtained a radiation probability [4] by using a model trajectory called the "th-trajectory"

$$\boldsymbol{\beta}_{\perp}(t) = \boldsymbol{b}_0 + \boldsymbol{b} \tanh\left(\frac{t}{T}\right), \tag{7}$$

where

$$\begin{aligned} \boldsymbol{b}_0 &= \frac{\boldsymbol{\beta}_{\perp 1} + \boldsymbol{\beta}_{\perp 2}}{2} \quad , \quad \boldsymbol{b} = \frac{\boldsymbol{\beta}_{\perp 2} - \boldsymbol{\beta}_{\perp 1}}{2}, \\ & \begin{cases} \boldsymbol{\beta}_{\perp 1} \equiv \boldsymbol{\beta}_{\perp}(t \to -\infty) \\ \boldsymbol{\beta}_{\perp 2} \equiv \boldsymbol{\beta}_{\perp}(t \to +\infty). \end{cases} \end{aligned}$$

The th-trajectory approaches to the free motion (i.e. straight paths) at $t \to \pm\infty$ while the trajectory is substantially bent at $|t| \lesssim T$ depending on the strength of the field. Since Eq.(7) is integrated analytically, we obtain an analytic expression for J:

$$J_{\text{th}} = \int_0^\infty \frac{dz}{z} \left[\nu^2 \left(\frac{1-\eta_+}{\eta_+} + \frac{\eta_+}{1-\eta_+} \right) \tanh^2 z - 1 \right] \sin\left[\tilde{\xi}(z - \mu \tanh z) \right] + \frac{\pi}{2}, \tag{8}$$

where $\nu = \gamma_+ b$ and $\tilde{\xi} = \nu(1+\nu^2)/[\eta_+(1-\eta_+)\chi]$.

3 Numerical results

A typical ν dependence of Eq.(8) is shown in Fig. 2. When ν becomes larger, the result of J_{th} approaches to J_{cf}. From Fig. 2 it is clear that the spectra become CFA-like as the angle decreases while Bethe-Heitler-like as the angle increases.

Based on Eq.(8), we have calculated the crystal-assisted pair production (CAPP) probability as a function of the photon incident angle in comparison with the experimental data by Belckacem et al.[5]. For simplicity, first we calculate the scattering angle by the impact approximation where the momentum change during the collision is proportional to the force multiplied by the "interaction time". The interaction time has been estimated by $a\rho/v_{\perp}$, where ρ is the impact parameter of the radiation point (for CAPP, we should have called it the "pair production point") and a is the parameter ($a \sim 1$). In Fig. 3 we have shown the results of the impact approximation. Taking account of the simplicity of the impact approximation, the agreement is satisfactory. J_{th} with the impact approximation will be useful for planning experiments.

In Fig. 4, a more involved calculation has been made by using thermally-averaged one-string Molière potential [7] at $T = 100$K. Though in this case we have no free parameter, the agreement is well. The peaks at higher photon energies look like somewhat

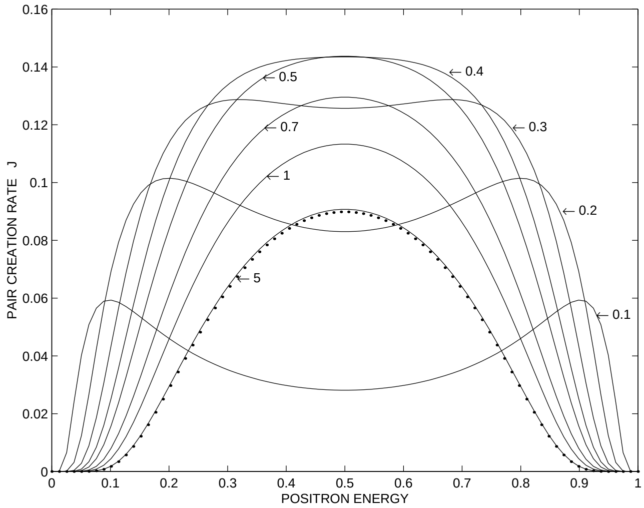

Figure 2: The ν-dependence of pair production probability calculated by using Eq.(8) as a function of the energy of produced positrons $E_+/\hbar\omega$. The dotted line represents the constant field approximation (CFA) of Eq.(6). ($\chi = 1$)

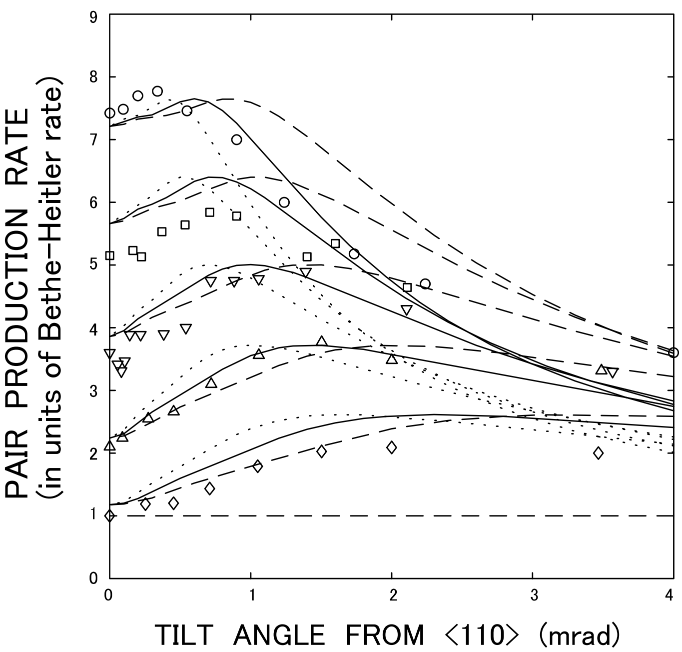

Figure 3: Comparison of the impact approximation results with the experimental data by Belkacem et al. [5]. The symbols mean the experimental results corresponding the energy of photons in the range of: ○: 150-120GeV, □:120-90GeV, ▽:90-60GeV, △:50-40GeV, ◇:40-20GeV. The dotted lines, solid lines, and broken lines correspond to $a = 2$, $a = 2\sqrt{2}$, and $a = 4$, respectively.

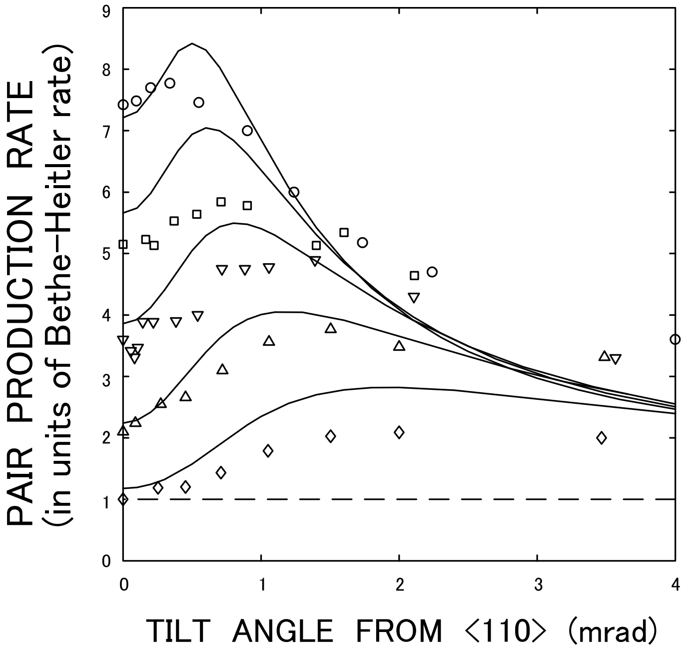

Figure 4: Comparison with experimental data by the rigorous scattering calculation with the use of the Molière one-string potential. No fitting parameter is included.

steeper than the experimental data, which may be due to neglecting of the many-string effect.

More detailed discussion will be published elsewhere.

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