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Semiclassical Theory of Crystal-Assisted Pair Production: Beyond the Uniform Field Approximation

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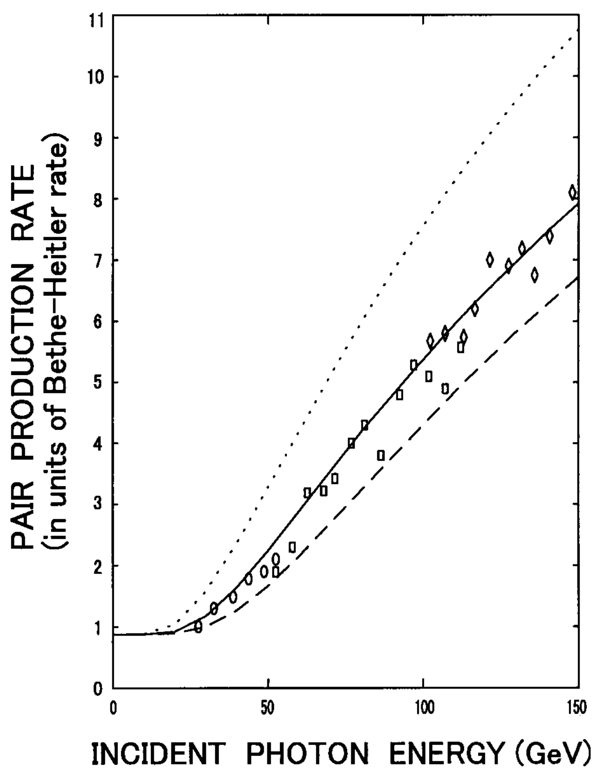


【Classical Radiation】

Schwinger(Phys.Rev.**75** 1912(1949))

$$\frac{dN}{d\omega}(t) = \frac{2\alpha}{\pi\gamma^2}\, J(t,\omega)$$

$$J = \int_0^\infty \gamma^2(1-\boldsymbol{\beta}_+\cdot\boldsymbol{\beta}_-)\sin\Delta_\mathrm{s}\frac{d\tau}{\tau} - \frac{\pi}{2} \tag{1}$$

$$\Delta_\mathrm{s} = \omega(\tau - |\boldsymbol{r}_+ - \boldsymbol{r}_-|/c)$$

where α is the fine structure factor, γ the Lorentz factor, $\boldsymbol{r}_\pm$,$\boldsymbol{\beta}_\pm$ the position and velocity at time $t\pm\tau/2$, respectively:

$$\begin{cases} t_+ = t+\tau/2 \\ t_- = t-\tau/2 \end{cases} \quad \begin{cases} \boldsymbol{\beta}_+ = \boldsymbol{\beta}(t_+) \\ \boldsymbol{\beta}_- = \boldsymbol{\beta}(t_-) \end{cases} \quad \begin{cases} \boldsymbol{r}_+ = \boldsymbol{r}(t_+) \\ \boldsymbol{r}_- = \boldsymbol{r}(t_-) \end{cases}$$

Decomposing the motion of the electron as longitudinal and transverse ones.

$$\begin{cases} \boldsymbol{r}(t) = \boldsymbol{\rho}(t) + \boldsymbol{z}(t) \\ \boldsymbol{\beta}(t) = \boldsymbol{\beta}_\perp(t) + \boldsymbol{\beta}_z(t) \end{cases}$$

Eq.(1) may be rewritten in the form that includes only the transverse motion:

$$J = \int_0^\infty \left[1 + \frac{\gamma^2}{2}(\delta\boldsymbol{\beta}_\perp)^2\right] \sin \Delta_\mathrm{s} \frac{d\tau}{\tau} - \frac{\pi}{2}$$

$$\Delta_\mathrm{s} = \frac{\omega\tau}{2\gamma} - \frac{\omega}{2c^2\tau}(\delta\boldsymbol{\rho})^2 + \frac{\omega}{2}\int_{t_-}^{t_+} \beta_\perp^2(\tau')d\tau' \qquad (2)$$

$$\begin{cases} \delta\boldsymbol{\beta}_\perp = \boldsymbol{\beta}_\perp(t_+) - \boldsymbol{\beta}_\perp(t_-) \\ \delta\boldsymbol{\rho} = \boldsymbol{\rho}(t_+) - \boldsymbol{\rho}(t_-) \end{cases}$$

By expanding factors in terms of τ, we obtain,

$$\Delta_\mathrm{s} = \frac{3}{2}\xi(x + x^3/3 + a_5x^5 + a_7x^7 + ...)$$

$$1 + \frac{\gamma^2}{2}(\delta\boldsymbol{\beta}_\perp)^2 = (1 + 2x^2 + b_4x^4 + b_6x^6 + ...)$$

where x and ξ are Schwinger's dimensionless time and frequency parameters, respectively:

$$x = \frac{g\gamma\tau}{2} \quad , \quad \xi = \frac{2\omega}{3g\gamma^3}.$$

g the acceleration divided by c. $a_5, a_7, ..., b_4, b_6, ...$ may be represented by the time derivatives of $\boldsymbol{g}$ and γ.

【Synchrotron Approximation】

If we approximate the instantaneous trajectory about $t = 0$ by a circular path, then we have $a_5, b_4 \sim 1/\gamma^2$, $a_7, b_6 \sim 1/\gamma^3 \cdots$. So, by neglecting the higher order terms under the condition $\gamma \gg 1$, we obtain the synchrotron formula:

$$J_{\text{syn}} = \frac{1}{\sqrt{3}} \left[2\mathrm{K}_{\frac{2}{3}}(\xi) - \int_{\xi}^{\infty} \mathrm{K}_{\frac{1}{3}}(\lambda) d\lambda \right] \qquad (3)$$

where $\mathrm{K}_n(x)$ is the modified Bessel functions.

If the trajectory of the electron is sufficiently a circular path at the radiation point, then the formula holds. However, if the trajectory becomes like a straight path, the circular path approximation does not hold because the neglected terms become larger.

【Standard th-trajectory】

A model trajectory for an arbitraly motion where at $t \to \pm\infty$ the velocity becomes constant while at $t = 0$ the acceleration is maximum (Khokonov and Nitta, Phys.Rev.Lett. **89** 094801 (2002)):

$$\boldsymbol{\beta}_\perp(t) = \boldsymbol{b}_0 + \boldsymbol{b}\tanh\left(\frac{t}{T}\right)$$

$$\boldsymbol{b}_0 = \frac{\boldsymbol{\beta}_{\perp 1} + \boldsymbol{\beta}_{\perp 2}}{2} \quad , \quad \boldsymbol{b} = \frac{\boldsymbol{\beta}_{\perp 2} - \boldsymbol{\beta}_{\perp 1}}{2} \tag{4}$$

$$\begin{cases} \boldsymbol{\beta}_{\perp -} \equiv \boldsymbol{\beta}_\perp(t \to -\infty) \\ \boldsymbol{\beta}_{\perp +} \equiv \boldsymbol{\beta}_\perp(t \to +\infty) \end{cases}$$

where T is the interaction time.
With this "th-trajectory" we obtain an analytic expression of radiation spectrum which has two parameters. From (2) and (4), we obtain,

$$J_{\text{th}} = \int_0^\infty (1 + 2\nu^2 \tanh^2 z) \sin \Delta_{\text{s}} \frac{dz}{z} - \frac{\pi}{2}$$

$$\Delta_{\text{s}} = \frac{3}{2}\xi\nu[(1+\nu^2)z - \nu^2 \tanh z] \tag{5}$$

$\nu \equiv \gamma b$ (non-dipole parameter)

【Quantum Correction】

The well known Baier-Katokov formula has been used for quantum correction. We have

$$\frac{dN}{d\eta} = \frac{\alpha c}{\pi \lambda_c} \frac{1}{\gamma} J \tag{6}$$

where λ_c is the Compton wavelength, $\eta = \hbar\omega/E$. J is given as

Synchrotron approximation

$$J_{syn} = \frac{1}{\sqrt{3}} \left[\left(1 - \eta + \frac{1}{1-\eta} \right) K_{\frac{2}{3}}(\xi^*) - \int_{\xi^*}^{\infty} K_{\frac{1}{3}}(\lambda) d\lambda \right] \tag{7}$$

th-trajectory approximation

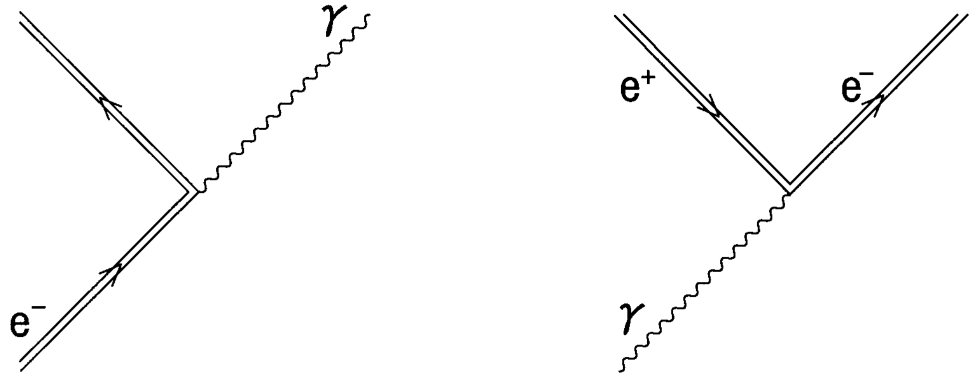
$$J_{th} = \int_0^{\infty} \left[1 - \nu^2 \left(1 - \eta + \frac{1}{1-\eta} \right) \tanh^2 z \right] \sin \Delta_s \frac{dz}{z} - \frac{\pi}{2}$$

$$\Delta_s = \frac{3}{2} \xi^* \nu \left[\left(1 + \nu^2 \right) z - \nu^2 \tanh z \right] \tag{8}$$

$$\begin{cases} \xi^* = 2\eta/3(1-\eta)\chi \\ \chi = \gamma \lambda_c F/mc^2 \end{cases} \tag{9}$$

where F is the transverse force from the field. We call χ invariant field parameter.

【Crossing Symmetry】



Once an expression of radiation is established, the pair production can be calculated by using the *crossing symmetry* of the matrix element.

By changing the variables as

$$\begin{cases} E \quad \text{(initial energy)} & \rightarrow \quad -E_\mathrm{p} \quad \text{(produced positron)} \\ E' \quad \text{(final energy)} & \rightarrow \quad E_\mathrm{e} \quad \text{(produced electron)} \\ \omega \quad \text{(emitted photon)} & \rightarrow \quad -\omega \quad \text{(absorbed photon)} \end{cases}$$

and multipling the rate of the density of final states to dN

$$\frac{E_\mathrm{p}^2 dE_\mathrm{p}}{(\hbar\omega)^2 d(\hbar\omega)}$$

we obtain the pair production probability for the th-trajectory approximation as well as the synchrotron approximation.

【Pair Production】

The pair production probability per unit time is given as

$$\frac{dN}{d\eta} = \frac{\alpha c}{\pi \lambda_{\mathrm{C}}} \frac{mc^2}{\hbar\omega} J^{\mathrm{pp}} \tag{10}$$

where $\eta = E^{(+)}/\hbar\omega$ (the energy ratio of the produced positron and the incident photon).

Uniform Field (Synchrotron) Approximation

$$J^{\mathrm{pp}}_{\mathrm{syn}} = \frac{1}{\sqrt{3}}\left[\left(\frac{1-\eta}{\eta} + \frac{\eta}{1-\eta}\right) \mathrm{K}_{\frac{2}{3}}(\xi^*) + \int_0^\infty \mathrm{K}_{\frac{1}{3}}(\lambda) d\lambda\right] \tag{11}$$

Th-trajectory Approximation

$$J^{\mathrm{pp}}_{\mathrm{th}} = \int_0^\infty \left[\nu^2 \left(\frac{1-\eta}{\eta} + \frac{\eta}{1-\eta}\right) \tanh^2 z - 1\right] \sin \Delta_{\mathrm{s}} \frac{dz}{z} + \frac{\pi}{2}$$

$$\Delta_{\mathrm{s}} = \frac{3}{2}\xi^* \nu \left[\left(1+\nu^2\right) z - \nu^2 \tanh z\right] \tag{12}$$

$$\xi^* = \frac{2}{3\eta(1-\eta)\chi}$$

non-dipole parameter

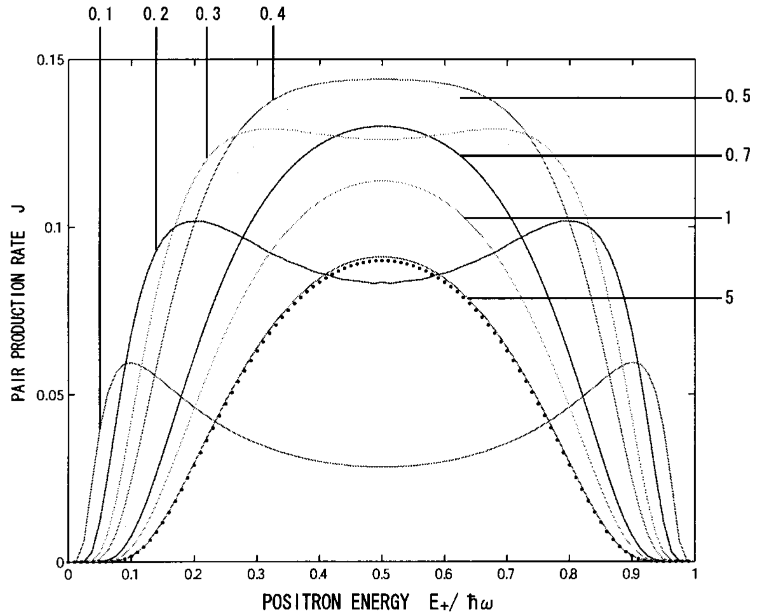
$\nu = \gamma_{\mathrm{p}} b \qquad \gamma_{\mathrm{p}}$: (positron's Lorentz factor)

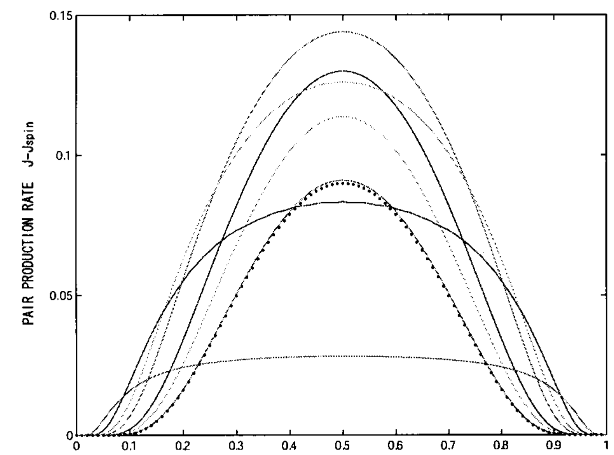
invariant field parameter

$$\chi = \frac{\hbar\omega\lambda_{\mathrm{C}} F}{m^2 c^4}$$

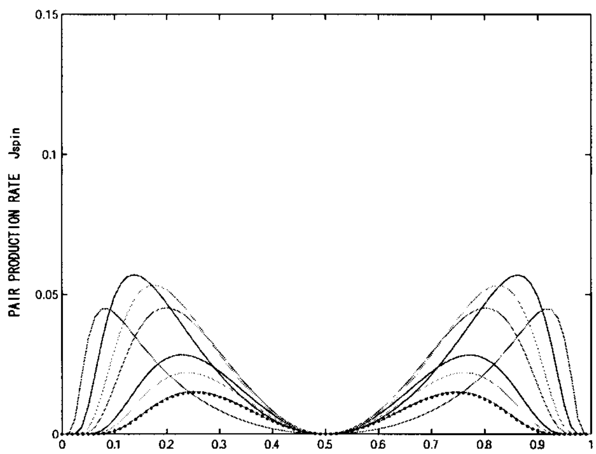
Pair production probability for $\chi = 1$. dotted line:UFA.

A: J_c. B: J_{spin} (spin-flip contribution)



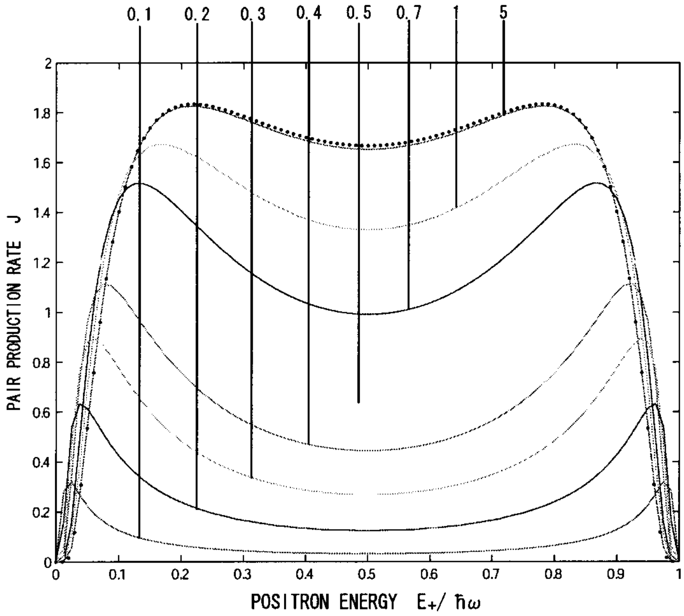


A

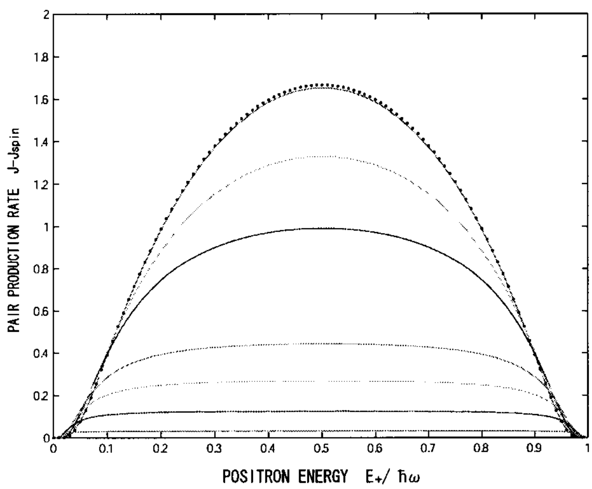


B

$\chi = 5$:

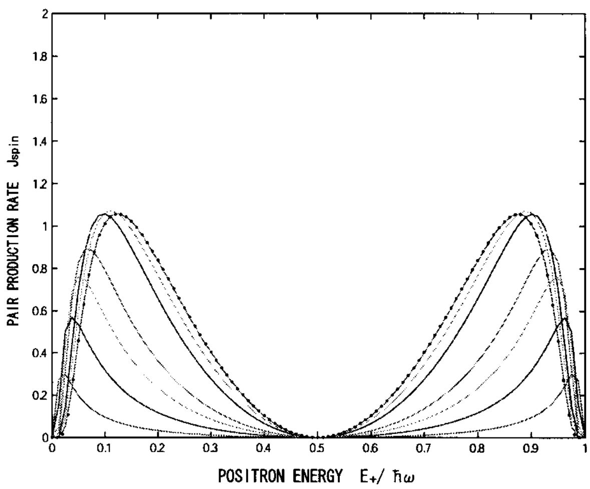


0.1
0.2
0.3
0.4
0.5
0.7
1
5
PAIR PRODUCTION RATE J
POSITRON ENERGY E+/ ħω



PAIR PRODUCTION RATE J-Jspin
POSITRON ENERGY E+/ ħω

A



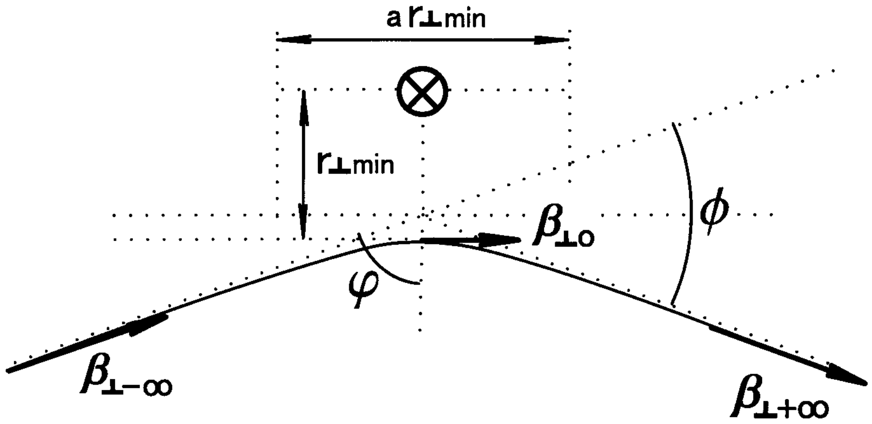
PAIR PRODUCTION RATE Jspin
POSITRON ENERGY E+/ ħω

B

【Impact Approximation】

non-dipole parameter ν is determined by the scattering angle.

$$\nu = \gamma_p \left| \frac{\vec{\beta}_{\perp+\infty} - \vec{\beta}_{\perp-\infty}}{2} \right| \tag{13}$$



For simplicity, we employ the impact approximation:

$$\gamma_{\mathrm{p}} mc\boldsymbol{\beta}_{\perp+\infty} - \gamma_{\mathrm{p}} mc\boldsymbol{\beta}_{\perp-\infty} = \boldsymbol{F}(\rho_0)\Delta t$$

where we have taken Δt as

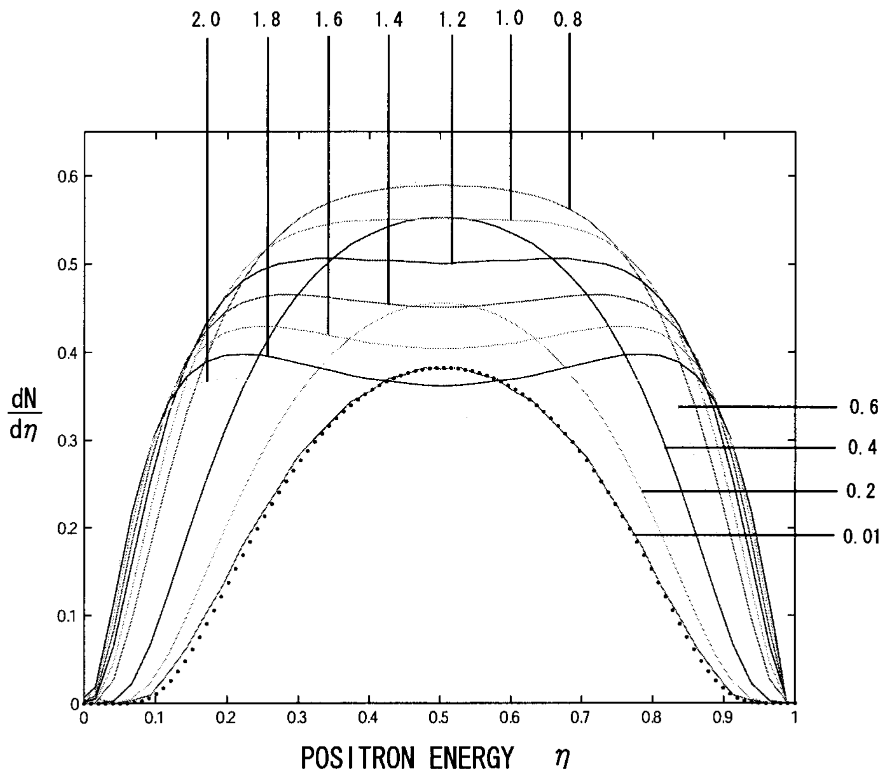
$$\Delta t = \frac{a\rho_0}{c\beta_{\perp 0}}$$

a is the fitting parameter: $a \sim 1$. By taking $\rho_0 = r_{\perp min}$, the non-dipole parameter becomes

$$\nu = \frac{a}{2} \frac{F(r_{\perp min})\, r_{\perp min}}{mc^2\theta_\gamma} \tag{14}$$

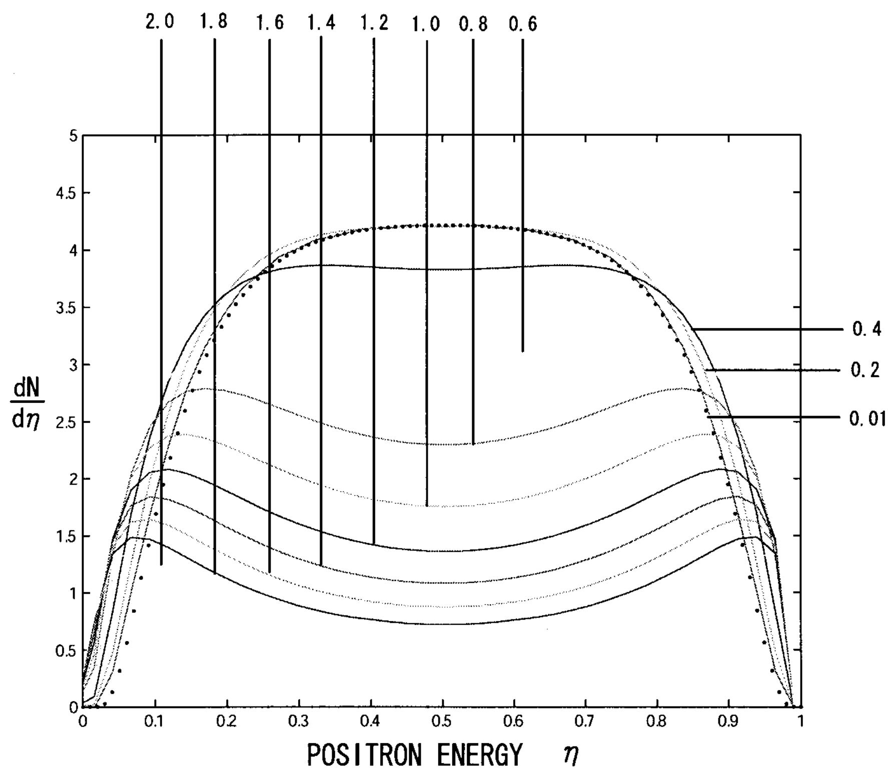
where θ_γ is the incident angle of the photon to the crystal axis.

Pair production spectra. The numbers represent the incident angle [mrad]. The dotted line shows UFA.
50[GeV] photon → Si< 110 >:

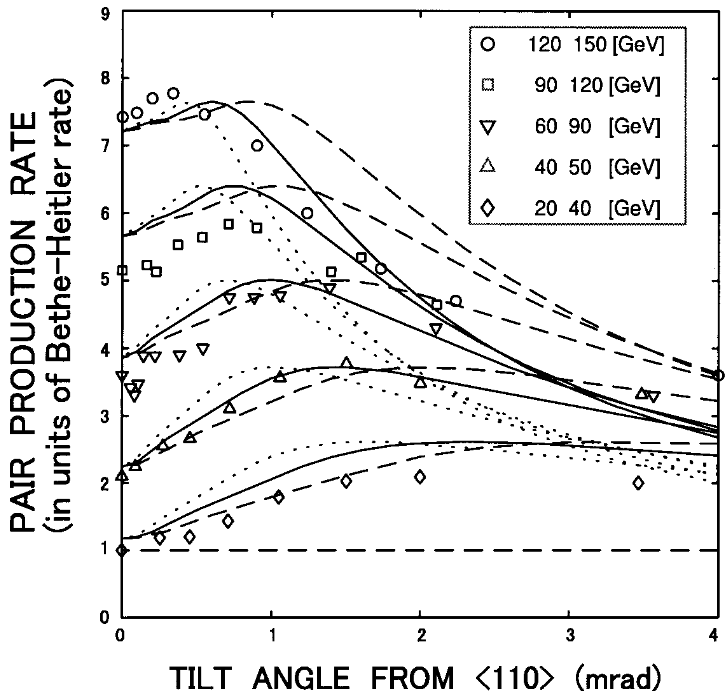


:

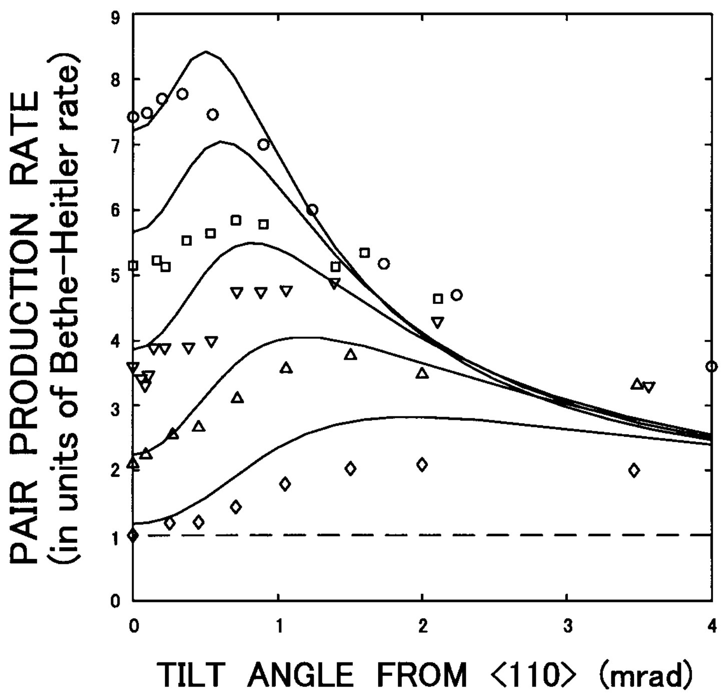
150[GeV]:



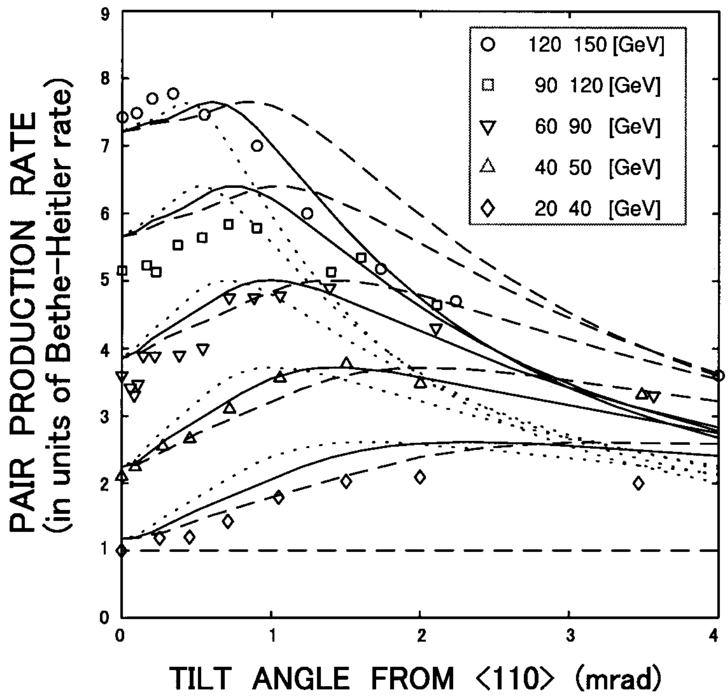
Impulse approximation:



One-string Molière potential ($T = 100$K):



Impulse approximation:



One-string Molière potential ($T = 100$K):